Applied Empirical Macroeconomics

Lecture 1: Stylized Macroeconomic Facts of the European and US Business Cycle

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Reading List

**Stylized Facts of Growth**

1. **Stylized Facts of economic growth:**

   - Empirical regularities characterizing the long run behavior of important macro aggregates, first labeled "stylized facts" by Nicholas Kaldor (1957).\(^1\)
   - The stylized facts of growth provide guidance on the choice of functional forms and parameter values in theoretical business cycle models (especially DSGE models).

   1. Labor productivity\(^2\) grows at a more or less constant rate
   2. Real output and the real capital stock grow at approximately the same rate (constant capital coefficient)
   3. The capital intensity of production grows at a more or less constant rate
   4. The rate of return in capital is approximately constant
   5. The income shares of capital and labor are approximately constant
   6. The growth rate of output per-capita varies substantially across countries

   - Facts (1), (2) and (4) imply facts (3) and (5).

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\(^1\)See also Cooley and Prescott (1995).
\(^2\)Defined as output per working hour.
2. Stylized Facts of the business cycle:

- Empirical regularities characterizing the cyclical behavior (fluctuations around the long run trend) of important macro aggregates.

- The stylized facts are the features of the data we would like a business cycle model to replicate.

- To separate trend and cycle, most people use the Hodrick-Prescott (HP) filter. This is an empirical method to extract a smooth trend from macroeconomic time series.

- To characterize empirical regularities, it is common to report measures of volatility (variance, standard deviation), persistence (autocorrelations) and co-movement (cross-correlations).
Stylized Facts of the Business Cycle

Important stylized facts:

1. **Volatility:** consumption in general slightly less volatile than output; investment about two to three times as volatile as output; employment and the Solow residual slightly less volatile than output.

2. **Correlation with output:** consumption, investment, employment and the Solow residual are strongly procyclical; net exports tend to be countercyclical; government consumption tends to be countercyclical or almost uncorrelated with output.

3. **Cross-country correlations:** correlations of output, consumption, investment, employment and the Solow residual across countries tend to be positive and sizable in magnitude; cross-country correlations regarding consumption slightly lower than that with respect to output.

4. Sizable cross-country heterogeneity with respect to the magnitude of the above statistics.

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3 Based on the assumption of a Cobb-Douglas technology for aggregate output: \( Y_t = Z_t N_t^{0.65} K_t^{0.35} \), the Solow residual is defined implicitly as

\[
\log(Z_t) = z_t = \log(Y_t) - 0.65 \log(N_t) - 0.35 \log(K_t) = y_t - 0.65n_t - 0.35k_t \rightarrow z_t \approx y_t - 0.65n_t,
\]

since the stock of capital exhibits negligible fluctuations over the business cycle.
Stylized Facts of the Business Cycle

3. Some comments:

▶ Fact (1): "investment more volatile than output, consumption less volatile than output"
Probably due to the attempt of agents to smooth consumption over time?

▶ Fact (2): "the Solow residual is strongly procyclical"
May be due to productivity shocks (shifts in the Solow residual) being an important driver of business cycle fluctuations?

▶ Fact (2): "net exports tend to be countercyclical"
This suggests that country specific shocks shifting output and the demand for imported goods in the same direction (e.g. demand side shocks) may be more important than shocks originating abroad which, everything else equal, shift home exports and home output in the same direction. Home productivity shocks leading to strong negative terms-of-trade effects may be also an important contributor to the negative correlation between net exports and output.4

4 As a consequence of a positive home productivity shock, the supply of home goods in the international goods market typically increases relative to foreigners’ demand for home goods. As a result, the terms of trade (defined as the price of home country’s exports relative to the price level of its imports) tend to worsen. The theory suggests that under certain conditions (especially for a low elasticity of substitution between home and foreign goods in private agents’ utility functions, home bias in consumption and incomplete financial markets, see Corsetti and Dedola (2005), Cole and Obstfeld (1991) and others) the terms of trade effect is strong enough to induce a deterioration of the home country’s trade balance. In other words, even though the volume of exported goods increases relative to the volume of imports the nominal value of the country’s exports falls relative to the value of its imports.
3. Some comments (continued):

▶ Fact (2): "government consumption tends to be either countercyclical or acyclical"
The negative correlation between government consumption (that include social security benefits) and output may be the result of governments attempting to stabilize the economy or simply letting the so called automatic stabilizers work. The near zero correlation between GDP and public consumption apparent in some countries may be due to substantial (implementation) lags in the government’s reaction to the state of the economy.

▶ Fact (3): "cross country correlation of consumption slightly lower than that of output"
Efficient risk sharing (or efficient hedging of country specific income and consumption risks) implies that the cross-country correlation of consumption should be substantially higher than that of output. Thus, this stylized fact may result from the incompleteness of financial markets\(^5\) disabling agents to insure against country specific fluctuations, or it may be simply due to the unwillingness\(^6\) of households and firms to share risks internationally.

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\(^5\)There are a lot of potential reasons for international financial markets to be incomplete: an insufficient number of internationally tradable securities, information inefficiencies, credit constraints, transaction costs, barriers to market entry and many others.

\(^6\)This may be due to certain combinations of individual agents’ preferences.
## Stylized Facts I

**Table:** Standard deviations and relative standard deviations, 1970:Q1 - 2006:Q4

<table>
<thead>
<tr>
<th>Country</th>
<th>$sd(y)$</th>
<th>$sd(nx)$</th>
<th>$sd(c)/sd(y)$</th>
<th>$sd(i)/sd(y)$</th>
<th>$sd(gov)/sd(y)$</th>
<th>$sd(n)/sd(y)$</th>
<th>$sd(z)/sd(y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.37</td>
<td>1.09</td>
<td>0.76</td>
<td>3.39</td>
<td>1.27</td>
<td>0.90</td>
<td>0.77</td>
</tr>
<tr>
<td>Austria</td>
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<td>0.70</td>
<td>1.10</td>
<td>2.70</td>
<td>0.54</td>
<td>0.68</td>
<td>0.98</td>
</tr>
<tr>
<td>Canada</td>
<td>1.47</td>
<td>0.92</td>
<td>0.80</td>
<td>2.77</td>
<td>0.76</td>
<td>0.80</td>
<td>0.67</td>
</tr>
<tr>
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<td>3.11</td>
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<td>0.63</td>
</tr>
<tr>
<td>Germany</td>
<td>1.20</td>
<td>0.68</td>
<td>0.98</td>
<td>2.91</td>
<td>0.98</td>
<td>0.71</td>
<td>0.84</td>
</tr>
<tr>
<td>Italy</td>
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</tr>
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<td>0.78</td>
<td>0.55</td>
<td>0.79</td>
</tr>
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<td>0.61</td>
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<td>2.58</td>
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<td>0.84</td>
<td>0.79</td>
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<tr>
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<td>2.84</td>
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<td>Euro Area</td>
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<td>0.84</td>
<td>2.44</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Legend:**
- $sd(x)$ - standard deviation of $x$; $y$ - log of GDP; $c$ - log of private consumption; $i$ - log of GFCF; $gov$ - log of government consumption; $n$ - log of employment; $z$ - log of the Solow Residual (cyclical component of Total Factor Productivity (TFP)); $nx$ - ratio of net exports to GDP: $nx = (Exp - Imp) / GDP$.

**Source:**
- OECD: Main Economic Indicators, IMF: International Financial Statistics, own calculations
## Stylized Facts II

**Table:** Contemporaneous correlations with GDP, 1970:Q1 - 2006:Q4

<table>
<thead>
<tr>
<th>Country</th>
<th>$cr(y, c)$</th>
<th>$cr(y, i)$</th>
<th>$cr(y, \text{gov})$</th>
<th>$cr(y, nx)$</th>
<th>$cr(y, n)$</th>
<th>$cr(y, z)$</th>
<th>$cr(y, \pi)$</th>
<th>$cr(y, r)$</th>
<th>$cr(y_t, y_{t-1})$</th>
</tr>
</thead>
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<td>Australia</td>
<td>0.44</td>
<td>0.76</td>
<td>0.16</td>
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<td>0.71</td>
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<tr>
<td>UK</td>
<td>0.78</td>
<td>0.66</td>
<td>-0.20</td>
<td>-0.30</td>
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<td>0.85</td>
<td>0.02</td>
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<td>0.94</td>
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<td>0.94</td>
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<td>-0.37</td>
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</tr>
</tbody>
</table>

**Legend:**

$cr(x_1, x_2)$ - correlation between $x_1$ and $x_2$.

**Source:**

OECD: Main Economic Indicators, IMF: International Financial Statistics, own calculations
Stylized Facts III

Table: Contemporaneous correlations with same US variable, 1970:Q1 - 2006:Q4

<table>
<thead>
<tr>
<th>Country</th>
<th>( c_\text{r}(y, y^{US}) )</th>
<th>( c_\text{r}(c, c^{US}) )</th>
<th>( c_\text{r}(i, i^{US}) )</th>
<th>( c_\text{r}(\text{gov}, \text{gov}^{US}) )</th>
<th>( c_\text{r}(n, n^{US}) )</th>
<th>( c_\text{r}(z, z^{US}) )</th>
<th>( c_\text{r}(\pi, \pi^{US}) )</th>
<th>( c_\text{r}(r, r^{US}) )</th>
</tr>
</thead>
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<td>Austria</td>
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<td>0.41</td>
<td>0.17</td>
<td>0.28</td>
<td>0.26</td>
<td>0.52</td>
<td>0.43</td>
</tr>
<tr>
<td>Canada</td>
<td>0.71</td>
<td>0.56</td>
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<td>0.64</td>
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<td>0.84</td>
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<tr>
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<td>0.27</td>
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<tr>
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<td>EU 15</td>
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<td>0.06</td>
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</tr>
</tbody>
</table>

Source: OECD: Main Economic Indicators, IMF: International Financial Statistics, own calculations
The Hodrick-Prescott Filter
Procedures to Extract a Trend from Macroeconomic Time Series

- Business cycle analysis is concerned with the fluctuations around the long run growth path.
- Problem: how can we separate trend and cycle?
- The usual solution is to apply a filter which “extracts” a smooth trend from the time series.
- This might be problematic if the underlying trend is not as smooth. However, assuming a smooth trend works well empirically.
- Filters proposed in the literature:
  - Linear/quadratic trend
  - Hodrick-Prescott filter (most popular filter)
  - Baxter-King filter
  - Christiano-Fitzgerald filter
  - Rotemberg filter
  - Unobserved components models
  - Beveridge-Nelson decomposition
The Hodrick-Prescott Filter (1)

The Hodrick-Prescott filter solves for the following minimization problem

\[ \min_{g_t} \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2, \]  

with \( \lambda > 0 \) as so called “smoothing parameter”, where \( y_t \) is the initial time series and \( g_t \) is the trend.
The Hodrick-Prescott Filter (2)

The smoothing parameter solves a trade off between

1. an approximation of the time series by the trend component, which is as exact as possible

\[ \sum_{t=1}^{T} (y_t - g_t)^2 \]  \hspace{1cm} (2)

2. and a trend, which is as smooth as possible

\[ \sum_{t=2}^{T-1} \left[ (g_{t+1} - g_t) - (g_t - g_{t-1}) \right]^2. \]  \hspace{1cm} (3)
The higher $\lambda$, the higher the punishment for a variation in the slope of the trend component.

For $\lambda \to \infty$, the extracted trend component tends towards a linear trend (which has constant slope).

Problem: $\lambda$ has to be given exogenously.

Important: Dependence of the periodicity of the data.

Hodrick and Prescott:
- Monthly frequency: $\lambda = 14400$
- Quarterly frequency: $\lambda = 1600$
- Annual frequency: $\lambda = 100$
The Hodrick-Prescott Filter (4)

Problems with the Hodrick-Prescott filter:

- **End point problem:** To calculate the trend component in period $t$, $g_t$, reliably, the filter needs a sufficient number of observations before and after this period. Therefore, the trend becomes more and more unreliable towards the sample end. As a remedy, it is suggested to extend the sample with forecasts. However, this typically mitigates the end point problem only somewhat, because the forecasts can be wrong. Empirically, the Hodrick-Prescott (and almost any other) filter is very volatile at the sample end.

- **Trend breaks:** The Hodrick-Prescott filter cannot account for structural breaks in the long term trend.

- **Smoothing parameter:** The Hodrick-Prescott is somewhat ad-hoc in the sense that the typical values for the smoothing parameter are not optimal in any setting.
GDP in Germany and HP Trend (Sample end: 2010 Q4)

Figure: Gross Domestic Product

Output Gap in Germany (Sample end: 2010 Q4)

Figure: Business Cycle Component of the Real Gross Domestic Product in Germany (HP-Trend)
The End Point Problem of the Hodrick-Prescott Filter

Figure: Gross Domestic Product

Output gap in quarter 1 2008, dependent on the time horizon of the HP filter applied:
HP-Trend Q2 2008: 1.41 %
HP-Trend Q2 2009: 3.26 %
HP-Trend Q2 2010: 3.76 %
HP-Trend Q4 2010: 3.69 %