Redistribution effects of flexible pension take-up

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Abstract

This paper explores the redistributive and welfare implications of a widely-applied pension reform aimed at increasing the flexibility of individual pension take-up. For that purpose, we use a two-period overlapping-generations model with a Beveridgean pay-as-you-go pension scheme and heterogeneous agents who differ in age, ability and life expectancy. Redistribution is driven by exogenous variation in life expectancy but also by endogenous schooling and retirement decisions. We find that introducing flexible pension take-up can induce a Pareto improvement if this reform is conducted in a proper way. Such a reform entails the application of uniform actuarial adjustment of pension entitlements based on the average life expectancy. Moreover, we argue that such a flexibility reform could further be improved if it incorporates actuarial non-neutral elements oriented to stimulate working.

Key words: redistribution, retirement, flexible pensions

JEL codes: H55, H23, J26

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1 Introduction

Since 1970, the effective retirement age of labour market exit has declined in almost all Western countries while life expectancy has increased substantially. Both developments have contributed to an increase of the average retirement period relative to the working period thereby eroding the fiscal sustainability of pension schemes. To reverse this trend, in recent years more attention has been given to pension reforms that improve labour supply incentives and encourage people to work longer. Indeed, most pension systems now allow for a flexible choice of the retirement age with more or less actuarially-neutral adjustments. This applies to public, occupational and individual pension schemes. This kind of reforms has the double advantage that it can increase the sustainability of pension systems but also reduce the distortions to the labour market caused by incentives to retire early. From 2005 onwards, people indeed stay longer in the labour market although effective retirement ages are currently still below the normal statutory pension age in the majority of OECD countries (OECD, 2011).

A main property of pension reforms in general and these flexibility reforms in particular is that they are typically implemented in a *uniform* way, applied to all participants. However, since individuals have heterogeneous socioeconomic characteristics (for example in terms of life expectancy or income level), uniformly implemented reforms may affect individual welfare in a rather different way. Indeed, it is well-known that pension schemes based on uniform policy rules contain large redistribution effects within and across generations, some intentional, and others unintentional (see e.g., Börsch-Supan and Reil-Held, 2001; ter Rele, 2007 and Bonenkamp, 2009). One of the main objectives of unfunded pension schemes, especially those of the Beveridgean type, is to redistribute income from high to low incomes. Apart from this, these pension schemes may also contain redistribution from short-lived to long-lived agents because they are typically based on collective annuities which do not depend on individual life expectancy. This makes collective annuities open to the objection that they lead to more regressive pension schemes because higher-income individuals tend to live longer.

Especially during periods of pension reforms, policy makers should consider to what extent the redistribution effects of pension schemes are still in line with their main objectives. Against this background, this paper explores the redistribution and welfare effects of pension reforms directed towards the introduction of a flexible pension take-up. To investigate this, we develop a two-period overlapping-generations model populated with heterogeneous agents who differ in age, ability and life expectancy. It is assumed that productivity of an individual is positively linked to life expectancy. When young, agents have to decide upon consumption, schooling and retirement. Born low-skilled, an agent can acquire extra skills and become high-skilled by investing time in schooling. Once high-skilled, an agent earns a higher wage rate during the remaining working period. As such, our model is consistent with empirical evidence which finds a strong positive association between longevity and socioeconomic status, either measured in terms of income or education level (see e.g., Pappas et al., 1993; Adams et al., 2003 or Meara et al., 2008). The benchmark pay-as-you-go (PAYG) social security system is of the Beveridgean type and is characterized by life-time annuities combined with inflexible pension take-up. In this way, the pension scheme includes both income redistribution (from rich to poor) and life-span redistribution (from short-lived to long-lived).

To implement a flexible pension take-up, policy makers have to determine how pension entitlements are adjusted if people postpone or advance retirement. We distinguish three scenarios which differ in the information set available to the government. In the first scenario, we assume that the government can observe individual life spans and uses this information to determine the actuarial adjustment factor. In the second scenario, the government is not able to observe individual life spans but it can observe skill levels which partly reveal information about life expectancy. In the third and last scenario, the government will not differentiate in the adjustment of benefits and simply relies on the average life expectancy of the total population. Using average life expectancy, the pension scheme will necessarily contain redistribution from short-lived to long-lived individuals. After all, these three scenarios share in common that the adjustment of benefits is at least actuarially neutral for some *average* individual.[1] We also explore the implications of retirement flexibility when it is combined with actuarial non-neutral adjustment directed at postponing retirement.

This paper relates to different strands of literature. It is closely connected to studies that analyse the interaction between pension schemes, schooling and retirement decisions (see e.g., Hougaard Jensen et al., 2003; Lau and Poutvaara, 2006 and Jacobs, 2009) and to a growing literature that focuses on the role of alternative pension systems when income and life expectancy are correlated (see e.g., Borck, 2007; Gorski et al., 2007; Hachon, 2008 and Cremer et al., 2010). In addition, our work is also inspired by Fisher and Keuschnigg (2010) and Jaag et al. (2010) who investigate the labour market impact of pension reforms towards more actuarial neutrality. Most of all these aforementioned studies focus on

[1] In this paper, we make a distinction between actuarial fairness and actuarial neutrality. *Actuarial fairness* requires that the present value of life-time contributions equals the present value of life-time benefits. *Actuarial neutrality* is a marginal concept and requires that the present value of accrued benefits for working an additional year is the same as in the year before (Queisser and Whitehouse, 2006).

pension reforms that strengthen the link between contributions and benefits. Our study, in contrast, mainly deals with more recent reforms aimed at introducing more flexibility in individual pension take-up. Apart from this, the methodological novelty of our paper is that it analyses the impact of pension reform on schooling, retirement decisions and intragenerational redistribution (originating from heterogeneity in ability as well as life expectancy) within one coherent framework.

From a policy perspective, this paper gives some interesting insights. First, we show that moving from a PAYG scheme with inflexible pension take-up to a pension scheme in which agents are free to choose the starting date of their pension income can induce a Pareto improvement. Such a reform would entail the application of uniform actuarial adjustments of pension entitlements (based on the average life expectancy) to ensure that high-skilled agents still reap the profits of redistribution from short- to long-lived people and are stimulated to postpone retirement. To compensate the low-skilled, a necessary condition is that the contribution rate is sufficiently high such that they benefit from more pensions enabled by the additional contributions of the high-skilled (who work longer). Moreover, we show that combining uniform adjustment with actuarial non-neutrality aimed at stimulating labour supply can further improve the reform and enables a Pareto improvement even at lower contribution rates. The intuition is that actuarial non-neutrality can partly undo the labour supply distortions associated with proportional taxes on labour income.

The key insights obtained in this paper are related to those in Cremer and Pestieau (2003). They show that increasing the effective retirement age, induced by a reduction in the implicit tax rate, generates a 'double dividend': it will free resources to meet the challenge of ageing and it could improve welfare of those with low wages and poor health. They identify as necessary conditions for such a double dividend that the benefit rule must both redistribute within generations and induce early retirement. Our pension scheme also satisfies these conditions and, when using uniform actuarial adjustment of benefits, a reform towards flexible pension take-up also results in this double dividend. Cremer and Pestieau (2003) obtain a reduction in the *implicit* tax rate by implementing *explicit* taxation that increases with age. In our model, in contrast, a reduction in implicit taxation follows from *uniform* actuarial adjustment of benefits in combination with a *positive* correlation between life expectancy and income. In this setting, individuals with high life expectancy (and thus high earnings) have an incentive to postpone retirement implying that most of the reform's cost is borne by those individuals without diminishing the welfare of the people with relatively low wages and life expectancies.

This paper is structured as follows. In Section 2 we introduce the basic model. This model contains a Beveridgean pension scheme with inflexible pension take-up and life-time annuities. Section 3 analyses the redistribution and welfare effects of a pension reform towards flexible take-up of pension benefits. In this section it is assumed that actuarial adjustment is actuarially neutral, at least for the 'average' individual. In Section 4 we deviate from this assumption and combine a flexible pension take-up with non-neutral actuarial adjustment of benefits. Section 5, finally, concludes the paper.

2 The benchmark model

We consider a two-period overlapping-generations model with a Beveridgean PAYG scheme and heterogeneous agents who differ in age, ability and life expectancy. In the first period of life, an individual decides whether to acquire skills or not and how much to save. In the second period, he decides which fraction of time he will spend on working and on enjoying retirement. The social security scheme offers a life-time annuity that starts paying out from the statutory retirement age until the end of life. Agents are allowed to continue their working life after the statutory retirement age or to advance retirement and stop working before the official retirement age. As a consequence, the statutory retirement is related to the date agents receive their pension benefit which is not necessarily equal to the *effective* retirement date.

2.1 Preferences

Suppose that preferences over first-period and second-period consumption are represented by the following utility function:

$$U(c,x) = u(c) + \pi u(x) \tag{1}$$

with $u'(\cdot) > 0$ and $u''(\cdot) < 0$; c is first-period consumption, x is second-period consumption and $\pi \leq 1$ is the length of the second period. The interest rate and the discount rate are zero.[2] Second-period consumption is defined net of the (monetary) disutility of labour:

$$x = \frac{d}{\pi} - \frac{\gamma}{2}\left(\frac{z}{\pi}\right)^2 \tag{2}$$

[2]Zero interest rate and zero discount rate are assumed for clarity sake. We also abstract from population growth, which implies that the internal rate of return of the PAYG scheme equals the interest rate so that we can concentrate on the intragenerational redistribution effects of the PAYG scheme. These assumptions could easily be relaxed. In numerical simulations we will allow for a positive discount rate.

where d is total consumption of goods when old yielding a consumption stream of d/π, z denotes the working period and γ is the preference parameter for leisure. Following Andersen (2005) we assume that disutility of working is related to the fraction of the second period spent on working (i.e., z/π). In addition, for analytical purposes, we take a quadratic specification (see also Cremer and Pestieau, 2003 or Cremer et al., 2010). This specification makes the problem more tractable at the cost that there are no income effects in labour supply. However, income effects in labour supply are found to be small when compared to substitution effects, see e.g., Blundell and MaCurdy (1999). Relating disutility of work to the fraction of the second period spent on working implies that for given retirement age an agent with a short life span experiences a higher disutility to work than an agent with a long life span.[3]

2.2 Innate ability and skill level

Following Razin and Sadka (1999), there are two levels of work skill, denoted by 'low' (L) and 'high' (H). Born low-skilled, an agent can acquire extra skills and become a high-skilled worker by investing $1-a$ units of time in schooling. The rest of the time, a, is devoted to working as a high-skilled worker.

The individual-specific parameter a reflects the ability of individuals to acquire working skills. The higher is a, the more able is the individual, and the less time a worker needs for acquiring a work skill. The parameter a ranges between 0 and 1 and its cumulative distribution function is denoted by $G(\cdot)$, that is $G(a)$ is the number of individuals with an innate ability parameter below or equal to a. We henceforth refer to an individual with an innate ability parameter of a as an a-individual. For the sake of simplicity, we normalize the number of individuals born in each period to be one, that is: $G(1) = 1$.

It is assumed that a high-skilled worker provides an effective labour supply of one unit per unit of working time. A low-skilled worker provides only $q < 1$ units of effective labour for each unit of working time. This difference between effective labour supply also applies to the second period of life. Let w denote the wage rate per unit of effective labour. Then the maximum amount of income agents can earn in the first period, denoted $W_y(a)$, depends on the skill level and is defined as:

$$W_y(a) \equiv \begin{cases} qw & \text{for} \quad a \leq a^* \\ aw & \text{for} \quad a \geq a^* \end{cases} \tag{3}$$

[3] If appropriate, we will discuss how model results change if disutility is related to the absolute working period z instead of the relative working period z/π.

with a^* the cut-off ability level to become high-skilled or low-skilled which will be determined later on. For the second period of life the maximum labour income, $W_o(a)$, equals:

$$W_o(a) \equiv \begin{cases} qw & \text{for} \quad a \leq a^* \\ w & \text{for} \quad a \geq a^* \end{cases} \tag{4}$$

2.3 Life expectancy

Each individual lives completely his first period of life (with a length normalized to unity) but only a fraction $\pi(a) < 1$ of his second period of life. We assume that $\pi'(a) \geq 0$: the higher the innate ability of an agent (i.e., higher a), the larger the length of life. In our model, the probability that agents with a high innate ability will opt for schooling is higher than agents with low ability levels. As a consequence, our model contains a positive correlation between life expectancy and schooling. Since the schooling decision increases the wage rate, the model is in line with the empirical evidence that wages positively co-move with life expectancy.[4]

Whenever necessary to parameterize the function $\pi(a)$, we will use the following specification:

$$\pi(a) = \bar{\pi}\left[1 + \lambda(a - \bar{a})\right], \quad \lambda \geq 0 \tag{5}$$

with $\bar{a} \equiv \int_0^1 a\,\mathrm{d}G$ the average ability level. This simple function has the following attractive properties. First, note that $\bar{\pi}$ represents the average life span in the economy. Second, there is a positive link between ability and the length of life governed by the parameter λ. Indeed, $\mathrm{Cov}(\pi, a) = \lambda \mathrm{Var}(a) > 0$. Third, life expectancy increases asymmetrically in the sense that high-ability people benefit (in absolute terms) more from an increase in the average life span than low-ability people, i.e., $\pi(a = 1) - \pi(a = 0) = \lambda\bar{\pi}$. This property is also consistent with empirical findings (Pappas et al., 1993; Mackenbach et al., 2003; Meara et al., 2008).

2.4 Consumption and retirement

An a-individual faces the following intertemporal budget constraint:

$$c + d = (1 - \tau)W_y(a) + (1 - \tau)zW_o(a) + P \tag{6}$$

[4]See Adams et al. (2003) for an extensive listing of studies dealing with the association of socioeconomic status and longevity.

where τ is a flat social security contribution (tax) rate; P are pension entitlements, equal to:

$$P = (\pi - \hat{z})b \tag{7}$$

with b the flat social security benefit per retirement period which starts at the statutory retirement age $\hat{z}$. As in most real-world social security schemes, tax contributions are proportional to the wage rate which reflects the objective of the scheme to redistribute income from rich to poor individuals.

Maximizing life-time utility (1) over c, d and z, subject to the life-time budget constraint (6) yields the following first-order conditions:

$$u'(c) = u'(x) \tag{8}$$

$$(1-\tau)W_o = \frac{\gamma z}{\pi} \tag{9}$$

Expression (8) is the standard consumption Euler equation. Equation (9) is the optimality condition regarding retirement and states that the marginal benefit of working (net wage rate) should be equal to the marginal cost of working (disutility of labour effect). From these first-order conditions, we obtain the following expressions for c and z:

$$c_{bev} = \frac{1}{1+\pi}\left[(1-\tau)W_y + \frac{(1-\tau)^2 W_o^2 \pi}{2\gamma} + (\pi - \hat{z})b\right] \tag{10}$$

$$z_{bev} = \frac{(1-\tau)W_o\pi}{\gamma} \tag{11}$$

where subscript 'bev' refers to the benchmark solution where we assume a Beveridgean PAYG scheme. Note that the social security tax distorts the retirement decision: the larger the contribution rate τ is, the faster agents leave the labour market, i.e., the smaller z, because it reduces the price of leisure. Notice further that our disutility specification ensures that the retirement period is proportional to longevity, i.e., $\pi - z = [1 - (1 - \tau)W_o/\gamma]\pi$, like in Andersen (2005). Hence, higher life expectancy is split between later retirement and a longer retirement period. Compared to high-skilled workers, low-skilled workers retire earlier for two reasons. First, since it is assumed that $q < 1$, low-skilled people have a lower wage rate (substitution effect). Second, low-skilled workers will generally have a lower life expectancy which induces them to leave the labour force earlier (disutility of labour effect).

2.5 Schooling

An agent is indifferent in acquiring skills or not if $U_L(a) = U_H(a)$.[5] Thus, there is a cut-off level of a, denoted a^* and given by:

$$a^*_{bev} = q - \frac{(1-\tau)w\pi(a^*_{bev})(1-q^2)}{2\gamma} \tag{12}$$

Agents with ability $a < a^*$ will not invest in schooling and stay low-skilled and agents with $a > a^*$ choose to acquire extra skills and become high-skilled. From this equation, we can infer the following results:

- An increase in the social security system (represented by an increase in τ) increases the fraction of low-skilled workers in the economy. That is,

$$\frac{\partial a^*}{\partial \tau} = \frac{w\pi(a^*)(1-q^2)}{2\gamma + (1-\tau)w\bar{\pi}\lambda(1-q^2)} > 0 \tag{13}$$

  The reason that the social security system affects schooling negatively is the endogenous retirement decision. With exogenous retirement the labour tax τ would not affect schooling, since all opportunity costs and benefits from investment in human capital receive a complete symmetric tax treatment (see e.g., Heckman, 1976). With endogenous retirement, the contribution tax rate induces agents to retire earlier which decreases the period in which schooling investments yield returns. As a result, the fraction of people for which schooling is profitable is lower.

- A general increase in life expectancy, governed by an increase in the parameter $\bar{\pi}$, raises the number of high-skilled individuals in the economy. That is:

$$\frac{\partial a^*}{\partial \bar{\pi}} = -\frac{(1-\tau)w(1+\lambda a^* - \lambda\bar{a})(1-q^2)}{2\gamma + (1-\tau)w\bar{\pi}\lambda(1-q^2)} < 0 \tag{14}$$

  Recall that an increase in the average life expectancy induces agents to postpone retirement. This increases the incentive to become high-skilled because the return period of schooling investment becomes longer.

- Decreasing the wage differential between skill levels, governed by an increase in the

[5] Throughout this paper, subscript 'L' refers to low-skilled workers and subscript 'H' refers to high-skilled workers.

parameter q, decreases the number of high-skilled workers:

$$\frac{\partial a^*}{\partial q} = \frac{\gamma + q(1-\tau)w\pi(a^*)}{\gamma + \frac{1}{2}(1-\tau)w\bar{\pi}\lambda(1-q^2)} > 0 \tag{15}$$

This makes sense, of course. If the reward of becoming high-skilled relative to staying low-skilled decreases, agents have a lower incentive to invest in skills. In the most extreme case when there is no reward of schooling ($q \to 1$), nobody will choose to acquire skills ($a^* \to 1$).

- The impact of life-span heterogeneity on schooling is ambiguous, because we have:

$$\frac{\partial a^*}{\partial \lambda} = \frac{(1-\tau)w\bar{\pi}(1-q^2)(\bar{a}-a^*)}{2\gamma + (1-\tau)w\bar{\pi}\lambda(1-q^2)} \gtreqqless 0 \quad \text{if} \quad a^* \lesseqqgtr \bar{a} \tag{16}$$

If the cut-off level is lower than the average ability level ($a^* < \bar{a}$), which means that the majority of the population is high-skilled, then the derivative is positive meaning that life-span heterogeneity reduces the incentive to acquire skills. The reason is that more heterogeneity (higher λ) reduces the life expectancy of those agents with $a < \bar{a}$. Hence, for agents just at the margin this lower life expectancy reduces the return period of schooling investment and therefore makes the option to stay low-skilled more attractive. Of course, if the cut-off level is higher than the average ability level ($a^* > \bar{a}$), we have the opposite situation meaning that more heterogeneity increases the number of high-skilled agents.

2.6 Social security

A feasible social security scheme must satisfy the following budget constraint:

$$b\int_0^1 (\pi - \hat{z})\,\mathrm{d}G = \tau q w \int_0^{a^*} (1 + z_L)\,\mathrm{d}G + \tau w \int_{a^*}^1 (a + z_H)\,\mathrm{d}G \tag{17}$$

This condition states that the total amount of pension benefits paid out (left-hand side) has to be equal to the total amount of tax contributions received (right-hand side).[6] Notice that the first term on the right-hand side are the tax payments of the low-skilled workers and the second term are the payments of the high-skilled workers. Using equa-

[6] We impose that $\pi - \hat{z} > 0$ for any a-individual. In other words, nobody passes away before the statutory retirement age.

tion (5), we have:

$$b(\bar{\pi} - \hat{z}) = \tau q w \int_0^{a^*} (1 + z_L)\, \mathrm{d}G + \tau w \int_{a^*}^1 (a + z_H)\, \mathrm{d}G \tag{18}$$

To indicate which agents are the net beneficiaries or net contributors of the redistributive pension scheme, we calculate the net benefit. The net benefit is the difference between the present value of pension benefits and tax contributions. If the present value of pension benefits exceeds pension contributions (positive net benefit), an agent is a net beneficiary. Otherwise, the agent is a net contributor. We can write net benefit (denoted NB) of high-skilled and low-skilled agents as:

$$NB_L \equiv (\pi - \hat{z})b - (1 + z_L)\tau q w \tag{19}$$

$$NB_H \equiv (\pi - \hat{z})b - (a + z_H)\tau w \tag{20}$$

A priori it is not immediately clear whether low-skilled or high-skilled agents are the net beneficiaries of the pension scheme. On the one hand, low-skilled agents have a lower wage rate and generally retire earlier than high-skilled agents. These factors imply that low-skilled agents benefit from the Beveridgean scheme. On the other hand, low-skilled agents enter the labour market earlier than high-skilled agents who spend part of the available working time on school. In addition, low-skilled agents die earlier than high-skilled agents. These two factors work in the opposite direction and imply that low-skilled agents are negatively affected by the pension scheme.

Because the interest rate is zero the implicit return of the PAYG scheme is the same as the market interest rate. From equations (18), (20) and (19) it therefore follows:

$$\int_0^{a^*} NB_L\, \mathrm{d}G + \int_{a^*}^1 NB_H\, \mathrm{d}G = 0 \tag{21}$$

This equation states that the sum of the net benefits of all (young) individuals is equal to zero which reflects the zero-sum game nature of the pension scheme.[7]

[7] With a positive interest rate the sum of net benefits would be negative because then all future generations have to pay for the windfall gain given to the old generation at the time the pension scheme has been introduced.

2.7 Liquidity constraint

Up to know, we have assumed that an agent can borrow against future pension income. In practice, though, this is almost impossible for reasons of moral hazard or adverse selection. Suppose we impose that agents are not able to frontload future pension income as a way to smooth consumption. More specifically, it is assumed that total consumption during the statutory retirement period cannot be lower than social security wealth. That is,

$$\frac{d}{\pi} \geq b \tag{22}$$

In terms of private savings, this condition implies:

$$s \geq -(1-\tau)zW_o + \hat{z}b \tag{23}$$

Agents cannot borrow more than the amount of human capital in the second period *adjusted* for the minimum consumption requirement which is related to the social security scheme.

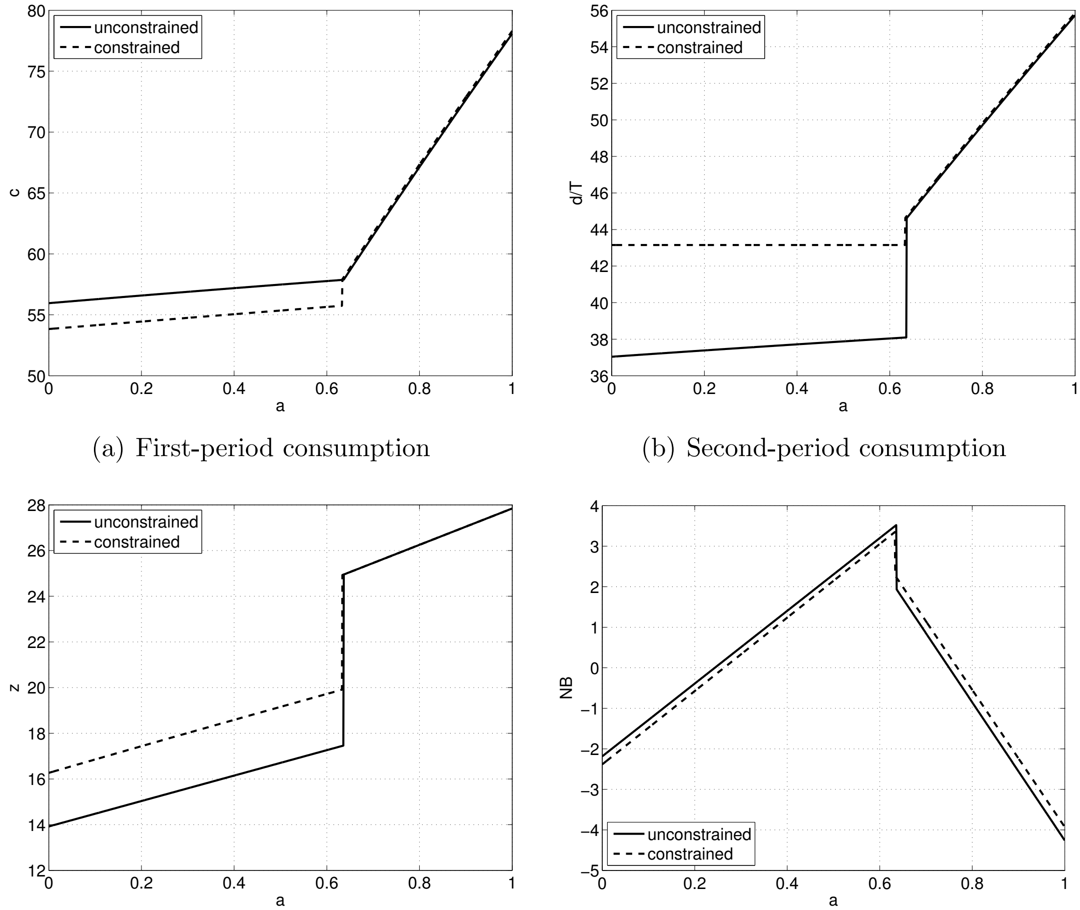
2.8 Numerical illustrations

To illustrate the mechanisms of the model, Figure 1 plots consumption (for young and old), retirement and net benefit as function of ability. These graphs are based on the following parameter values: $\tau = 0.25$, $w = 1$, $\gamma = 2.2$ and $\beta = 0.55$ where β denotes the time discount factor. Assuming that a model period lasts 30 years, a time discount factor of $\beta = 0.55$ corresponds to an annual time preference rate of 2%. We have $\hat{z} = 1/6$ and $\bar{\pi} = 0.7$ which is consistent with an official retirement age of 65 and an average life span of 81 year.[8] The heterogeneity parameter λ is calibrated such that the difference between the life span of the most and least productive agents roughly amounts to 7 years which is consistent with recent Dutch estimates. This gives $\lambda = 1/3$. For the wage differential we take $q = 0.7$ which implies that roughly one-third of the working population is high-skilled (CBS). Finally, we assume that ability a follows a uniform distribution, i.e., $G(a) = a$, and that the utility function is logarithmic, i.e., $u(\cdot) = \ln(\cdot)$.

Figure 1 shows the benchmark results for the liquidity unconstrained case (solid lines) and the constrained case (dashed lines). The size of the constrained group very much depends on the time preference rate. Given our parameterization, all low-skilled agents

[8]Life time consists of 30 years childhood (including primary and secondary education) that are not accounted for, 30 years of full potential working time (which can partly be used for tertiary education), and a last period of 30 years. Hence, the retirement age is $60 + 30z$ and the average life span is $60 + 30\bar{\pi}$.

Figure 1: The benchmark model



(a) First-period consumption

(b) Second-period consumption

(c) Working period

(d) Net benefit

Notes: Consumption and net benefit are expressed in percentage of the wage rate; the working period is expressed in percentage of the maximum period length (normalized to unity).

are liquidity constrained and all high-skilled agents are unconstrained. The constrained group is forced to transfer consumption from the first to the second period and to postpone retirement. Notice that agents with both a relatively low and high ability level experience a negative net benefit. The low-ability group suffers from the adverse redistribution from long- to short-lived agents. The high-ability group suffers from the redistribution from high- to low-income earners. Including a liquidity constraint decreases the net benefit of the constrained group because these agents have to work longer and thus contribute more taxes to the pension scheme. The net benefit of the unconstrained group increases, though, because the additional tax receipts of the constrained group raises the pension benefit.

3 Pension flexibility reforms

In this section, we consider the welfare and redistribution effects of different pension reforms aimed to introduce flexibility in the pension entitlement age. Throughout the analysis, we assume that the age at which individuals stop working equals the age at which pension outlays start. This is a realistic assumption because in many countries agents are simply not allowed to continue working once they opt for pension benefits.

When allowing for flexible pension take-up, policy makers have to decide upon the actuarial adjustment of benefits. There are numerous ways to do this. Here we focus on three scenarios which differ with respect to the information set available to the government. The first scenario is based on the (rather strong) assumption that the government is able to observe individual life expectancies. The government then sets individual-specific adjustment factors as a way to get rid of adverse redistribution from short- to long-lived agents. In the second, more realistic, scenario, the government is not able to observe individual life spans but it can observe skill levels because in the model there is a one-to-one relation between the wage rate and skill level. Skill levels partly reveal private information about life expectancies because only agents with relatively high ability (and therefore high longevity) will become high-skilled. In the third scenario, the government is not able (or not willing) to differentiate actuarial adjustment to individuals or socioeconomic groups and simply relies on the average life expectancy. A necessary implication of uniform actuarial adjustment is that it introduces adverse selection effects in the retirement decision. As will be seen, long-lived agents have an incentive to postpone retirement while short-lived agents have an incentive to advance retirement.

3.1 Actuarial adjustment of benefits

Suppose that the pension fund pays benefits p to the individual over its effective retirement period. Hence, total pension entitlements are equal to $P = (\pi - z)p$. Pension earnings per retirement period p are equal to:

$$p = m(z, \hat{\pi})b \tag{24}$$

where b is the reference flat pension benefit independent of contributions and labour history. The factor $m(\cdot)$ is the actuarial adjustment factor which determines to what extent the reference benefit b will be adjusted when agents retire later (or earlier) than the statutory retirement age. More specifically, it is defined in the following way:

$$m(z,\hat{\pi}) = \frac{\bar{\pi} - \hat{z}}{\hat{\pi} - z} \tag{25}$$

Notice that the adjustment factor is equal to the ratio between the 'average' retirement period ($\bar{\pi} - \hat{z}$) and the 'individual' retirement period measured by the reference life expectancy parameter $\hat{\pi}$ (to be specified below). Obviously, at the individual level, actuarial non-neutrality arises if $\hat{\pi}$ differs from $\pi(a)$ for an a-individual. The function $m(\cdot)$ is a positive function in the individual retirement decision z: if an agent decides to continue work after the official retirement age the pension benefit in the remaining retirement periods will be adjusted upward.

As explained, we consider three different cases for life expectancy to be used in the adjustment factor. First, in Section 3.2 we assume that actuarial adjustment occurs using individual life spans ($\hat{\pi} = \pi$). Second, in Section 3.3 the adjustment factor becomes skill dependent. In this case, the adjustment factor for the low-skilled agents will be based on the average life span of the low-skilled group ($\hat{\pi} = \bar{\pi}_L$) and that for the high-skilled agents will be based on the average life expectancy of the high-skilled people ($\hat{\pi} = \bar{\pi}_H$), where $\hat{\pi}_L$ and $\hat{\pi}_H$ are defined by:

$$\hat{\pi} = \begin{cases} \bar{\pi}_L \equiv \int_0^{a^*} \frac{\pi}{G(a^*)}\,\mathrm{d}G & \text{if} \quad a < a^* \\ \bar{\pi}_H \equiv \int_{a^*}^{1} \frac{\pi}{1-G(a^*)}\,\mathrm{d}G & \text{if} \quad a > a^* \end{cases} \tag{26}$$

Finally, in Section 3.4 we impose that adjustment is done in a uniform way based on the average life expectancy of the total population ($\hat{\pi} = \bar{\pi}$).

3.2 Individual actuarial adjustment of benefits

Suppose that individual life expectancies are observable. Then the government can use this information in the actuarial adjustment factor.

Actuarial adjustment factor

With individual adjustment, $\hat{\pi} = \pi(a)$. Then the adjustment factor m and pension entitlements P are equal to:

$$m = \frac{\bar{\pi} - \hat{z}}{\pi - z} \tag{27}$$

$$P = (\bar{\pi} - \hat{z})b \tag{28}$$

Note from equation (27) that $m = 1$ for an agent with an average ability level ($a = \bar{a}$) who retires at the statutory retirement age $\hat{z}$. For this so-called 'average' individual the pension benefit per retirement period is equal to the reference benefit, i.e., $p = b$. If this person retires later than the official retirement age, then $m > 1$, which means that he receives a per-period benefit which is adjusted upward, i.e., $p > b$. On the other hand, if the person retires earlier, we have $m < 1$ implying $p < b$.

With individual adjustment the retirement decision is actuarially neutral in the sense that the effective retirement date z has no effect on the total pension entitlement in the Beveridgean system. To see this, note that:

$$\frac{\partial P}{\partial z} = 0 \tag{29}$$

Hence, agents cannot increase their total pension entitlements by postponing or moving up retirement. Compared to the benchmark scheme with a fixed pension take-up at the official retirement age, individual actuarial adjustment removes redistribution related to life-span differences. Any individual, irrespective of life expectancy, income or skill level, receives exactly the same amount of pension benefits over the life time.

Consumption and welfare effects

Compared to the benchmark social security model, the retirement and schooling decisions are the same. Also the aggregate budget constraint of the pension contract does not change and, consequently, also the pension benefit per retirement period stays the same. The only thing in the model that changes are the consumption decisions which can be written as:

$$c_{ind} = c_{bev} + \frac{(\bar{\pi} - \pi)b}{1 + \pi} \tag{30}$$

From equation (30) we immediately infer the following result.

Proposition 1. *Introducing retirement flexibility using individual actuarial adjustment of pension benefits implies that the welfare of the short-lived agents ($\pi < \bar{\pi}$) increases while the welfare of the long-lived agents ($\pi > \bar{\pi}$) decreases.*

3.3 Skill-dependent actuarial adjustment of benefits

Policy makers who are mainly interested in removing adverse redistribution effects in pension schemes, would like to use individual actuarial adjustment. As shown in the previous section, actuarial adjustment based on individual life expectancies completely removes the adverse transfers from short- to long-lived agents. However, an important practical impediment of such reform is that individual life expectancies are often difficult to observe.

A possible solution to this asymmetric information problem is to base actuarial adjustment on characteristics which are observable and at least to some extent are correlated with individual life expectancies (see, e.g., Bovenberg et al., 2006). In terms of our model, this characteristic could be the skill level of agents. By choosing to become high-skilled or low-skilled, agents partly reveal information about their life expectancy. Indeed, life expectancy of high-skilled agents is on average higher than that of low-skilled agents. The government can use this information by conditioning the actuarial adjustment factor on skill level. This also reduces the redistribution from low- to high-skilled people.

Actuarial adjustment factor and budget constraint

With skill-dependent adjustment, the reference life expectancy measure is conditional on skill group: $\hat{\pi} = \bar{\pi}_L$ for the low-skilled group and $\hat{\pi} = \bar{\pi}_H$ for the high-skilled group. The actuarial adjustment factor is:

$$m = \begin{cases} \frac{\bar{\pi}-\hat{z}}{\bar{\pi}_L - z_L} & \text{if} \quad a < a^* \\ \frac{\bar{\pi}-\hat{z}}{\bar{\pi}_H - z_H} & \text{if} \quad a > a^* \end{cases} \tag{31}$$

and pension entitlements are equal to:

$$P = \begin{cases} \frac{(\pi - z_L)(\bar{\pi}-\hat{z})b}{\bar{\pi}_L - z_L} & \text{if} \quad a < a^* \\ \frac{(\pi - z_H)(\bar{\pi}-\hat{z})b}{\bar{\pi}_H - z_H} & \text{if} \quad a > a^* \end{cases} \tag{32}$$

From equation (31) it follows that skill-dependent adjustment reduces redistribution from short-lived to long-lived ability groups, like with individual actuarial adjustment. Indeed, suppose that all agents retire at the official retirement date $\hat{z}$. Then we have for the low-skilled group that $m > 1$ while for the high-skilled group $m < 1$. Hence, low-skilled agents are compensated for the fact that they have a lower life expectancy. However, contrary to individual adjustment, skill-dependent adjustment does not remove redistribution completely. As a consequence, these transfers will lead to distortions in the retirement decision. To see this, from equation (32) we have:

$$\Psi \equiv \frac{\partial P(z)}{\partial z} = \begin{cases} \frac{(\pi - \bar{\pi}_L)p}{\bar{\pi}_L - z_L} & \text{if} \quad a < a^* \\ \frac{(\pi - \bar{\pi}_H)p}{\bar{\pi}_H - z_H} & \text{if} \quad a > a^* \end{cases} \tag{33}$$

Since the amount of pension entitlements depends on the individual retirement age, skill-dependent actuarial adjustment introduces selection effects in the retirement decision. Notice that within a skill group, the conversion factor of agents with high life expectancies (with $\pi > \hat{\pi}$) is too high from an actuarial point of view. This stimulates them to postpone retirement because this increases their pension entitlements ($\Psi > 0$). For short-lived people (with $\pi < \hat{\pi}$) it is just the opposite; for these agents the conversion factor of continued activity is too low which stimulates early pension take-up. For these people postponing retirement would simply mean that pension entitlements decrease ($\Psi < 0$).

With skill-dependent actuarial adjustment, the budget constraint of the pension scheme equals:

$$\begin{aligned} b(\bar{\pi} - \hat{z}) \int_0^{a^*} \left(\frac{\pi - z_L}{\bar{\pi}_L - z_L} \right) \mathrm{d}G + b(\bar{\pi} - \hat{z}) \int_{a^*}^1 \left(\frac{\pi - z_H}{\bar{\pi}_H - z_H} \right) \mathrm{d}G = \\ \tau q w \int_0^{a^*} (1 + z_L) \, \mathrm{d}G + \tau w \int_{a^*}^1 (a + z_H) \, \mathrm{d}G \end{aligned} \tag{34}$$

where the first (*second*) term on the left-hand side denotes the total pension benefits of the low-skilled (*high-skilled*) people and the first (*second*) term on the right-hand side denotes the total tax contributions of the low-skilled (*high-skilled*) people.

Consumption and retirement

Compared to individual actuarial adjustment, a flexibility reform based on skill-dependent adjustment has more impact in the model. As we will show, this reform leads to different consumption, retirement and schooling decisions than in the benchmark system. In addition, these behavioural changes will also affect the budget constraint of the pension scheme so that pension benefits change.

With flexible pension take-up and skill-dependent actuarial adjustment, the life-time budget constraint of the a-individual equals:

$$c + d = (1 - \tau) W_y(a) + (1 - \tau) z W_o(a) + P(z) \tag{35}$$

with $P(z)$ already defined in equation (32). The first-order conditions are now given by:

$$u'(c) = u'(x) \tag{36}$$

$$(1 - \tau) W_o + \Psi(z) = \gamma \frac{z}{\pi} \tag{37}$$

with $\Psi(z)$ given by equation (33). Consumption and retirement are then equal to:

$$c_{edu} = c_{bev} + \frac{1}{1+\pi}\left[P_{edu} - P_{bev} - \frac{\Psi^2(z_{edu})\pi}{2\gamma}\right] \tag{38}$$

$$z_{edu} = z_{bev} + \frac{\Psi(z_{edu})\pi}{\gamma} \tag{39}$$

Observe from equation (39) that retirement behaviour is now subject to two labour supply distortions. Like before, we have that the contribution tax rate negatively affects retirement behaviour (through its impact on z_{bev}). Apart from this, the retirement decision is also distorted due to the redistribution effects as represented by $\Psi(z)$. This redistribution distortion works in both directions: it can either stimulate retirement or depress retirement, dependent on the difference between the individual life expectancy and the average life expectancy of the skill group to which the individual belongs. For individuals with below-average life spans ($\pi < \hat{\pi}$), we have $\Psi(z) < 0$ implying that they advance retirement. If individuals have above-average life spans ($\pi > \hat{\pi}$), then $\Psi(z) > 0$ meaning that they have an incentive to postpone retirement.

What about consumption under skill-dependent actuarial adjustment? Compared to benchmark consumption, note that equation (38) contains the term $-\frac{\Psi^2(z)\pi}{2\gamma}$. This term is negative and reflects the utility loss because of the redistribution distortion in the retirement decision. Of course, in principle flexibility can also induce a utility gain because an agent can choose the retirement age which gives him the highest pension entitlement. This potential gain is captured by the term $P_{edu} - P_{bev}$. Note from equations (7) and (32) that total pension benefits are generally not the same in the benchmark scheme and in the flexibility reform with skill-dependent adjustment.

Schooling

Skill-dependent actuarial adjustment changes schooling because it introduces an endogenous link between the schooling decision and the actuarial adjustment factor. Indeed, if agents choose to become high-skilled the reference life expectancy used in the conversion factor is different (and higher) from the one if they choose to stay low-skilled. We have the following result.[9]

Proposition 2. *Introducing retirement flexibility with skill-dependent actuarial adjustment of pension benefits increases the fraction of low-skilled agents in the economy. That*

[9]For analytical tractability, all analytical results derived in Section 3.3 and 3.4 assume that heterogeneity in individual life spans is sufficiently small, i.e., λ close to zero. Using numerical illustrations, it will be shown that our results do not hinge on this assumption.

means,

$$a^*_{edu} > a^*_{bev}$$

with a^*_{edu} the threshold value of education in case of skill-dependent actuarial adjustment.

Proof. The cut-off point is determined by the condition $U_H(a^*) = U_L(a^*) \Rightarrow c_H(a^*) = c_L(a^*)$. From equations (38) we can infer:

$$a^*_{edu} = a^*_{bev} - \frac{2\gamma\Theta}{(1-\tau)w\left[2\gamma + (1-\tau)w(1-q^2)\bar{\pi}\lambda\right]} \tag{40}$$

with,

$$\Theta \equiv P(z_H(a^*)) - P(z_L(a^*)) - \frac{\Psi^2(z_H(a^*))\pi(a^*)}{2\gamma} + \frac{\Psi^2(z_L(a^*))\pi(a^*)}{2\gamma}$$

From equation (40), we derive:

$$\left.\frac{\partial a^*_{edu}}{\partial\lambda}\right|_{\lambda=0} = \frac{\partial a^*_{bev}}{\partial\lambda} - \frac{1}{(1-\tau)w}\frac{\partial\Theta}{\partial\lambda} \tag{41}$$

From the definition of Θ it follows:

$$\left.\frac{\partial\Theta}{\partial\lambda}\right|_{\lambda=0} = \frac{(\bar{\pi}-\hat{z})b}{\bar{\pi}-z_H}\left[\frac{\partial\pi(a^*)}{\partial\lambda} - \frac{\partial\bar{\pi}_H}{\partial\lambda}\right] - \frac{(\bar{\pi}-\hat{z})b}{\bar{\pi}-z_L}\left[\frac{\partial\pi(a^*)}{\partial\lambda} - \frac{\partial\bar{\pi}_L}{\partial\lambda}\right] \tag{42}$$

From equations (5) and (26), we obtain:

$$\left.\frac{\partial\pi(a^*)}{\partial\lambda}\right|_{\lambda=0} = \bar{\pi}(a^* - \bar{a}) \tag{43}$$

$$\left.\frac{\partial\bar{\pi}_L}{\partial\lambda}\right|_{\lambda=0} = \frac{\int_0^{a^*}\bar{\pi}(a-\bar{a})\,\mathrm{d}G}{G(a^*)} \tag{44}$$

$$\left.\frac{\partial\bar{\pi}_H}{\partial\lambda}\right|_{\lambda=0} = \frac{\int_{a^*}^1\bar{\pi}(a-\bar{a})\,\mathrm{d}G}{1-G(a^*)} \tag{45}$$

Substituting these expressions in equation (42) and rearranging, gives:

$$\left.\frac{\partial\Theta}{\partial\lambda}\right|_{\lambda=0} = \underbrace{\frac{(\bar{\pi}-\hat{z})b}{\bar{\pi}-z_L}\frac{\int_0^{a^*}\bar{\pi}(a-a^*)\,\mathrm{d}G}{G(a^*)}}_{<0} - \underbrace{\frac{(\bar{\pi}-\hat{z})b}{\bar{\pi}-z_H}\frac{\int_{a^*}^1\bar{\pi}(a-a^*)\,\mathrm{d}G}{1-G(a^*)}}_{>0} < 0 \tag{46}$$

Hence, an increase in λ reduces Θ implying that $a^*_{edu} > a^*_{bev}$. □

With educational-specific actuarial adjustment, individuals can self-select the actuarial adjustment factor with their schooling decision. If they choose to become high-skilled this reduces *ceteris paribus* the adjustment factor of their pension benefits because this is now based on the average life expectancy of the high-skilled part of the population. Hence, individuals just at or around the margin will find it less attractive to become high-skilled.

Welfare effects

What does this flexibility reform imply for welfare of high-skilled and low-skilled ability groups? Is it possible that this reform induces a Pareto improvement? Obviously, for this to be the case, no agent should be worse off after the reform and at least one agent should be strictly better off.

Suppose that the reform takes place unexpectedly. How will this affect utility of the currently old generation? If the reform would not take place, second-period consumption would equal:

$$\pi x_{bev} = s + \frac{(1-\tau)^2 W_o^2 \pi}{2\gamma} + P_{bev} \tag{47}$$

After the reform, the first-order condition for the retirement decision of the old generation is given by equation (37). Using this condition, old-age consumption after the reform equals:

$$\pi x_{edu} = s + \frac{(1-\tau)^2 W_o^2 \pi}{2\gamma} + P_{edu} - \frac{\Psi(z)^2 \pi}{2\gamma} \tag{48}$$

The old generation is better off after the reform if $u(x_{edu}) - u(x_{bev}) \geq 0$, implying:

$$\pi x_{edu} - \pi x_{bev} \geq 0 \;\Rightarrow\; P_{edu} - P_{bev} - \frac{\Psi(z)^2 \pi}{2\gamma} \geq 0 \tag{49}$$

What about the young and future generations? Young and future generations are better off if $c_{edu} \geq c_{bev}$ for each ability level. From equation (38) it turns out that the condition for young and future generations is exactly the same as that for the old generation. Hence, when condition (49) is satisfied and holds strictly for at least one a-individual, the reform is Pareto improving. However, we can prove that this can never be the case as mentioned in the following proposition:

Proposition 3. *A pension reform from inflexible Beveridgean pensions towards flexible Beveridgean pensions with skill-dependent actuarial adjustment of pension benefits cannot be a Pareto-improvement.*

Proof. We have the following condition for a Pareto improvement:

$$\Gamma \equiv P_{edu} - P_{bev} - \frac{\Psi(z)^2\pi}{2\gamma}$$
$$= \frac{(\pi - z)(\bar{\pi} - \hat{z})}{\hat{\pi} - z} b_{edu} - (\pi - \hat{z}) b_{bev} - \frac{\Psi(z)^2\pi}{2\gamma} \geq 0 \tag{50}$$

We derive from this expression:

$$\left.\frac{\partial\Gamma}{\partial\lambda}\right|_{\lambda=0} = \frac{z - \hat{z}}{\bar{\pi} - z}\bar{\pi}(a - \bar{a}) b_{edu} + (\bar{\pi} - \hat{z}) \left(\frac{\partial b_{edu}}{\partial\lambda} - \frac{\partial b_{bev}}{\partial\lambda}\right) - \frac{\bar{\pi} - \hat{z}}{\bar{\pi} - z}\frac{\partial\hat{\pi}}{\partial\lambda} b_{edu} \tag{51}$$

To prove that this reform cannot be a Pareto-improvement, we have to show that for at least one a-individual equation (51) is strictly negative. Let us concentrate on a high-skilled agent with ability level $a = a^*$. Using equation (26), equation (51) then becomes:

$$\begin{aligned}\left.\frac{\partial\Gamma(a = a^*)}{\partial\lambda}\right|_{\lambda=0} = & \underbrace{\frac{z_H(a^*) - \hat{z}}{\bar{\pi} - z_H(a^*)}\bar{\pi}(a^* - \bar{a}) b_{edu}}_{\Sigma_1} + (\bar{\pi} - \hat{z}) \left(\frac{\partial b_{edu}}{\partial\lambda} - \frac{\partial b_{bev}}{\partial\lambda}\right) \\ & - \underbrace{\frac{\bar{\pi} - \hat{z}}{\bar{\pi} - z_H(a^*)} \frac{\int_{a^*}^1 \bar{\pi}(a - \bar{a})\,\mathrm{d}G}{1 - G(a^*)} b_{edu}}_{\Sigma_2 > 0}\end{aligned} \tag{52}$$

Because $\bar{\pi} > z_H(a)\ \forall a$ it follows that $|\Sigma_1| < \Sigma_2$. Hence, we certainly know that $\frac{\partial\Gamma(a=a^*)}{\partial\lambda} < 0$ if $b_{edu} < b_{bev}$ which means that the flexibility reform cannot be a Pareto-improving. In Appendix A.1 it is shown that indeed $b_{edu} < b_{bev}$ for any value of τ. $\square$

Hence, with the current flexibility reform it is not possible to achieve a Pareto improvement. Because this reform removes (at least to some extent) the redistribution from short-lived to long-lived individuals, it is not beneficial for high-skilled agents who generally have a higher life expectancy. In this respect, the welfare effects of skill-dependent adjustment are comparable with those of individual adjustment.

3.4 Uniform actuarial adjustment of benefits

In practice, individual life spans or skill levels are often difficult to observe. Therefore, in most real-world pension schemes actuarial adjustment is done in a uniform way based on some average life expectancy index.

Actuarial adjustment factor and budget constraint

With uniform adjustment, the reference life expectancy measure is the same for each ability group, $\hat{\pi} = \bar{\pi}$, so that the adjustment factor and pension entitlements are:

$$m = \frac{\bar{\pi} - \hat{z}}{\bar{\pi} - z} \tag{53}$$

$$P = \frac{(\pi - z)(\bar{\pi} - \hat{z})b}{\bar{\pi} - z} \tag{54}$$

Now $m = 1$ for each individual who retires at the statutory retirement age, so that $p = b$; agents who retire later than $\hat{z}$ receive a higher benefit, $p > b$, and agents who retire earlier receive less, $p < b$.

Contrary to individual adjustment or (to a lesser extent) skill-dependent adjustment, uniform adjustment does not remove the adverse redistribution from short-lived agents to long-lived agents. From equation (54) we observe that agents with long life spans receive more pension entitlements than agents with short life spans, *ceteris paribus*. This redistribution implies that the pension scheme is no longer actuarially neutral at the individual level. Indeed, from equation (54) it follows:

$$\Psi(z) \equiv \frac{\partial P(z)}{\partial z} = \frac{(\pi - \bar{\pi})p}{\bar{\pi} - z} \tag{55}$$

Comparable with skill-dependent adjustment, agents with above-average life spans ($\pi > \bar{\pi}$) have an incentive to delay retirement and agents with below-average life spans ($\pi < \bar{\pi}$) want to retire earlier. However, these selection effects will be higher than with skill-dependent adjustment because the heterogeneity of life expectancy in the total population is obviously higher than the life-span heterogeneity within skill groups.

In case of uniform actuarial adjustment the budget constraint of the pension scheme changes into:

$$\begin{aligned} b(\bar{\pi} - \hat{z}) \int_0^{a^*} \left(\frac{\pi - z_L}{\bar{\pi} - z_L} \right) \mathrm{d}G + b(\bar{\pi} - \hat{z}) \int_{a^*}^{1} \left(\frac{\pi - z_H}{\bar{\pi} - z_H} \right) \mathrm{d}G = \\ \tau q w \int_0^{a^*} (1 + z_L)\, \mathrm{d}G + \tau w \int_{a^*}^{1} (a + z_H)\, \mathrm{d}G \end{aligned} \tag{56}$$

where (again) the first (*second*) term on the left-hand side denotes the total pension benefits of the low-skilled (*high-skilled*) people and the first (*second*) term on the right-hand side represents the total tax contributions of the low-skilled (*high-skilled*) people.

Consumption and retirement

With uniform actuarial adjustment, consumption and retirement have exactly the same form as with skill-dependent actuarial adjustment. That is,

$$c_{uni} = c_{bev} + \frac{1}{1+\pi}\left[P_{uni} - P_{bev} - \frac{\Psi^2(z_{uni})\pi}{2\gamma}\right] \tag{57}$$

$$z_{uni} = z_{bev} + \frac{\Psi(z_{uni})\pi}{\gamma} \tag{58}$$

The only difference arises in the specification of $\Psi(z)$ which is now based on the average life expectancy of the total population, see equation (55).

Like with skill-dependent actuarial adjustment, retirement behaviour is again subject to two different labour supply distortions: the contribution rate τ negatively affects retirement behaviour and the derivative $\Psi(z)$ causes distortions in the retirement decision due to redistribution effects associated with life-span heterogeneity. Notice that the impact of this second distortion is larger than under skill-dependent actuarial adjustment because for the majority of the people the difference between the own life expectancy and the population-average life expectancy is larger than the difference between the own life expectancy and the skill-group average.

Schooling

A priori it is not immediately clear whether uniform actuarial adjustment will increase or decrease the incentives to become high-skilled. For long-lived agents who are stimulated to postpone retirement one expects that the willingness to become high-skilled increases as the return period of schooling investments is higher. On the other hand, for short-lived individuals it is more reasonable that the reform reduces schooling incentives. We can indeed derive the following result.

Proposition 4. *Introducing retirement flexibility with uniform actuarial adjustment of pension benefits will decrease the fraction of low-skilled agents if the minority of the population is high-skilled. Otherwise, the pension reform decreases the incentive to become high-skilled. That means,*

$$a^*_{uni} \gtreqless a^*_{bev} \quad \text{if} \quad a^*_{uni} \lesseqgtr \bar{a}$$

with a^*_{uni} the threshold value of education in case of uniform adjustment.

Proof. The cut-off point is determined by the condition $U_H(a^*) = U_L(a^*) \Rightarrow c_H(a^*) =$

$c_L\,(a^*)$. From equation (57) we can infer:

$$a^*_{uni} = a^*_{bev} - \frac{2\gamma\Theta}{(1-\tau)w\left[2\gamma + (1-\tau)w(1-q^2)\bar{\pi}\lambda\right]} \tag{59}$$

with,

$$\Theta \equiv P(z_H(a^*)) - P(z_L(a^*)) - \frac{\Psi^2(z_H(a^*))\pi(a^*)}{2\gamma} + \frac{\Psi^2(z_L(a^*))\pi(a^*)}{2\gamma}$$

From equation (59), it follows:

$$\left.\frac{\partial a^*_{uni}}{\partial \lambda}\right|_{\lambda=0} = \frac{\partial a^*_{bev}}{\partial \lambda} - \frac{1}{(1-\tau)w}\frac{\partial \Theta}{\partial \lambda} \tag{60}$$

where we have used that $\Theta = 0$ if $\lambda = 0$. Using equations (54) and (55), we derive from the definition of Θ:

$$\begin{aligned} \left.\frac{\partial \Theta}{\partial \lambda}\right|_{\lambda=0} &= \frac{\bar{\pi}(\bar{\pi}-\hat{z})b(a^*-\bar{a})(z_H - z_L)}{(\bar{\pi}-z_H)(\bar{\pi}-z_L)} \\ &= \frac{\gamma(\bar{\pi}-\hat{z})b(a^*-\bar{a})(1-\tau)w(1-q)}{(\gamma - w + \tau w)(\gamma - qw + \tau q w)} \end{aligned} \tag{61}$$

where we have used equation (58) in going from the first to the second line. Substituting equation (61) into equation (60), we ultimately obtain:

$$\left.\frac{\partial a^*_{uni}}{\partial \lambda}\right|_{\lambda=0} = \frac{\partial a^*_{bev}}{\partial \lambda} - \frac{\gamma(\bar{\pi}-\hat{z})b(a^*_{uni}-\bar{a})(1-q)}{(\gamma - w + \tau w)(\gamma - qw + \tau q w)} \tag{62}$$

From this equation it directly follows that $a^*_{uni} > a^*_{bev}$ if $a^*_{uni} < \bar{a}$, $a^*_{uni} = a^*_{bev}$ if $a^*_{uni} = \bar{a}$ and $a^*_{uni} < a^*_{bev}$ if $a^*_{uni} > \bar{a}$. $\square$

If the marginal agent has an above-average life span, $\pi(a^*) > \bar{\pi}$, this agent has an incentive to postpone the retirement decision when actuarial adjustment occurs in a uniform way. This increases the incentive to become high-skilled because later retirement raises the return period of schooling investments. On the contrary, when the marginal agent has a below-average life expectancy, $\pi(a^*) < \bar{\pi}$, this person has an incentive to advance retirement which decreases the willingness to become high-skilled.

In quantitative terms, the differences in educational attainment are small though. Note that the numerator of the last term of equation (62) is relatively small compared to the denominator. Simulations later on will confirm this observation.

Welfare effects

What are the individual welfare implications of reforming social security from a Beveridgean scheme without a flexible entitlement age towards a scheme with a flexible entitlement age based on uniform adjustment? For this reform to be a Pareto improvement, exactly the same condition should be satisfied as with skill-dependent adjustment. That is,

$$P_{uni} - P_{bev} - \frac{\Psi(z)^2\pi}{2\gamma} \geq 0 \tag{63}$$

with $\Psi(z)$ now defined by equation (55).

Assumption 1. *The statutory retirement age is set equal to the retirement decision of the individual with the average ability level, i.e., $\hat{z} = z(\bar{a})$.*

Analysing the possibility for a Pareto improvement, Assumption 1 is crucial and easy to understand. Individuals with below-average life expectancy have an incentive to advance retirement because from an actuarial point of view the adjustment factor of retirement postponement is too low for them. Therefore, for these people retiring *after* the statutory retirement age is not in their interest as it reduces pension entitlements compared to the benchmark *ceteris paribus*. For individuals with above-average life expectancy we just have the opposite. These individuals have an incentive to postpone retirement because the actuarial adjustment factor is too high for them. Hence, retiring *before* the statutory retirement is not in their interest.

Proposition 5. *Suppose Assumption 1 is satisfied. Then we have the following:*

i) There exists a tax rate τ^* for which it holds that a pension reform from inflexible Beveridgean pensions towards flexible Beveridgean pensions with uniform actuarial adjustment of pension benefits is a Pareto improvement.

ii) If $a^*_{uni} - a^*_{bev} \to 0$, the tax critical rate τ^* is uniquely determined and equal to:[10]

$$\tau^* = \frac{(\gamma - qw)\sqrt{\gamma - w} - (\gamma - w)\sqrt{\gamma - qw}}{w\sqrt{\gamma - qw} - qw\sqrt{\gamma - w}} \tag{64}$$

Then the reform is Pareto improving if and only if $\tau \geq \tau^*$.

[10] Note from Proposition 4, equation (62), that $a^*_{uni} - a^*_{bev} \to 0$ is a reasonable assumption.

Proof. We again have the following condition for a Pareto-improvement:

$$\begin{aligned}\Gamma &\equiv P_{uni} - P_{bev} - \frac{\Psi(z)^2\pi}{2\gamma} \\ &= \frac{(\pi - z)(\bar{\pi} - \hat{z})}{\bar{\pi} - z} b_{uni} - (\pi - \hat{z}) b_{bev} - \frac{\Psi(z)^2\pi}{2\gamma} \geq 0 \end{aligned} \tag{65}$$

where for at least one a-individual this inequality has to hold strictly. We start again from a situation in which each agent has the same life expectancy, i.e., $\lambda = 0 \Rightarrow \pi(a) = \bar{\pi}$ for each a-individual. Then $\Gamma = 0$. Now we increase $\lambda > 0$ and derive the following derivative:

$$\left.\frac{\partial \Gamma}{\partial \lambda}\right|_{\lambda=0} = \frac{z - \hat{z}}{\bar{\pi} - z}\bar{\pi}(a - \bar{a}) b_{uni} + (\bar{\pi} - \hat{z})\left(\frac{\partial b_{uni}}{\partial \lambda} - \frac{\partial b_{bev}}{\partial \lambda}\right) \tag{66}$$

Note that Assumption 1 implies that the minimum of the first term is equal to zero, i.e., for the agent with ability $a = \bar{a}$. Hence, the reform is Pareto-improving if $b_{uni} \geq b_{bev}$. In Appendix A.2 it is shown that this condition is satisfied if the tax rate is sufficiently high. □

Flexible pension take-up drives up pension costs, *ceteris paribus*, because the adverse selection effects induce people to retire at the date that gives them the highest pension. For given tax revenues, this implies that the pension benefit per retirement year should be lower than in the absence of these adverse selection effects. However, because uniform actuarial adjustment stimulates high-skilled agents to postpone retirement, these agents also contribute more taxes to the pension scheme. Therefore, if the contribution tax rate is sufficiently high, it is possible that the additional pension contributions paid by the high-skilled suffice to provide a higher flat pension benefit to everyone.

The selection effects due to uniform actuarial adjustment increase the adverse redistribution from low-skilled (short-lived) to high-skilled (long-lived) individuals. Consequently, to ensure that the reform is also beneficial for the low-skilled people, this disadvantage should be compensated by an increase in the benefit. As explained above, this can only be realized if the tax rate is sufficiently high. Recall from equation (11) and equation (58) that a switch from the benchmark scheme (with inflexible pension take-up) to a scheme with flexible pension take-up (combined with uniform actuarial adjustment) increases labour supply of the high-skilled people and decreases labour supply of the low-skilled. In this way, the tax rate is an effective instrument to increase income redistribution from rich to poor with this reform.

This result differs sharply from the reform discussed in the previous section. With skill-dependent actuarial adjustment, we have seen that the high-skilled agents suffer from the abolishment of the redistribution related to life-span heterogeneity. In that case, low-skilled agents are not able to compensate the loss of the high-skilled agents. As shown by Proposition 3, $b_{edu} < b_{bev}$ for any value of the contribution tax rate.

Proposition 5 shows that a pension reform towards flexible pension take-up in combination with uniform actuarial adjustment induces a double dividend. It reduces the implicit tax rate faced by the high-income individuals stimulating them to postpone retirement. This will free resources which, when there is a demographic shock, can (partly) be used to meet the increase in the dependency ratio. In addition, if the reform is conducted properly, it will also foster redistribution from rich to poor. Similar to Cremer and Pestieau (2003), this double dividend hinges on two conditions: the pension contract needs a downward distortion, the removal of which brings additional resources. The contract also needs to be redistributive from rich to poor so that most of the reform's cost is borne by individuals with relatively high earnings.

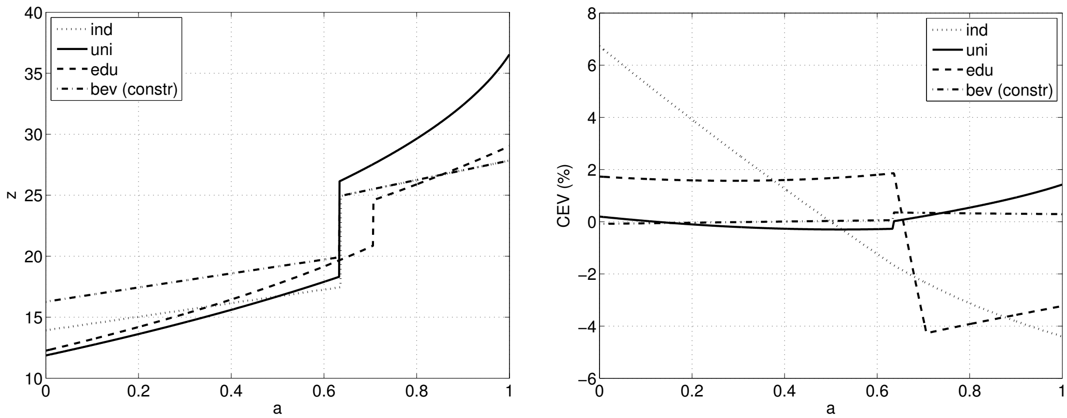
3.5 Numerical illustrations

Figure 2 shows retirement behaviour (left panel) and the welfare effects (right panel) of the different flexibility reforms. The parameter values used are the same as those in Figure 1. We measure welfare changes in consumption-equivalent variation (CEV): we ask what percentage of extra consumption an agent would require in the benchmark situation (without liquidity constraints) to be as well off as in the flexibility reform. Positive (*negative*) numbers thus indicate welfare gains (*losses*) from the reform.

Obviously, flexible pension take-up removes the liquidity constraint which is binding for the low-skilled agents in the benchmark scheme (recall Figure 1c). Therefore, in all flexibility reforms considered, low-skilled agents can afford to retire earlier than in the benchmark model. Notice that a switch towards uniform actuarial adjustment gives strong incentives to high-skilled agents to postpone retirement because this increases their pension entitlements. As the minority of the population is high-skilled, the reform slightly increases the fraction of high-skilled agents in the economy (see Proposition 4). The most pronounced schooling effects occur if actuarial adjustment is conditional on skill level. In this case, the fraction of high-skilled agents sharply declines because, once high-skilled, individuals will be confronted with a lower adjustment factor (see Proposition 2).

Individual welfare is mostly affected by a reform towards flexible pension take-up based on individual actuarial adjustment. This would imply that redistribution from short- to long-lived individuals disappears completely. With this reform, welfare of the least productive agents increase with almost 7% while that of the most productive agents

Figure 2: Flexibility reforms: retirement and welfare



(a) Retirement (b) Welfare

Notes: The working period is expressed in percentage of the maximum period length (normalized to unity); consumption-equivalent variation (CEV) is expressed in percentage of life-time consumption.

declines with more than 4%. With skill-dependent adjustment, we also have that welfare of the low-skilled (*high-skilled*) agents increases (*decreases*) because this reform lowers the adverse redistribution effects related to life-span heterogeneity.

Notice that a tax rate of 25% is not sufficient to ensure that uniform actuarial adjustment is Pareto improving: for most of the low-skilled agents welfare is lower than in the benchmark. To really achieve that the reform is Pareto improving, the contribution rate should almost be doubled to 48%, see Figure 3 (dashed line).

4 Flexibility and stimulating labour supply

In this final section, we show that a Pareto improvement of flexible pension take-up can also be achieved at lower contribution rates. The key driver behind this result is the introduction of incentives that stimulate voluntary postponement of retirement. One natural way to do this is rewarding retirement postponement at an actuarially non-neutral way.

In recent years, penalties and rewards for earlier or later retirement have increased in a number of countries. In general, the penalty rate has not been as high as the reward rate to stimulate work continuation. In the US, for example, for each year of retirement before the normal age, the annual benefit is reduced by 6.75%. The actuarial increment for those retiring after the normal age amounts to 8%. In Japan, the difference is even larger; there the penalty rate of early retirement is 6% per year while the reward rate of later retirement is 8.4% (OECD, 2011). Despite these policy measures to stimulate work, in most countries the current reductions of early pension benefits do still not fully correspond both to the lower amount of contributions paid by the worker and to the increase in the period over which the worker will receive pension payments (Queisser and Whitehouse, 2006).

4.1 Actuarial adjustment factor and budget constraint

To show that flexible pensions can be used to stimulate labour supply in the most stylized setting, we abstract in this section from heterogeneity in life expectancy. Hence, each agent, irrespective of ability, lives a fraction $\pi \leq 1$ of the second period. Suppose that the actuarial adjustment factor now has the following specification:

$$m(z, \pi) = \left(\frac{\pi - \hat{z}}{\pi - z} \right)^{\sigma} \qquad \sigma > 1 \tag{67}$$

The parameter σ governs the degree of actuarial non-neutrality. If $\sigma = 1$, adjustment is actuarially neutral with respect to the retirement decision (see previous section). If $\sigma > 1$, the adjustment factor is *higher* than the actuarially-neutral level if agents retire *later* than the statutory retirement age ($z > \hat{z}$). However, the adjustment factor is *lower* than the actuarially-neutral level if agents retire *earlier* than the official retirement age ($z < \hat{z}$). That means, specification (67) rewards postponing retirement and discourages early retirement.

Another way to illustrate this point, is to figure out how pension entitlements P will change if people postpone (or advance) retirement. Given equation (67), the pension entitlements are equal to:

$$P = (\pi - \hat{z})^{\sigma} (\pi - z)^{1-\sigma} b \tag{68}$$

Taking the derivative of P with respect to z then gives:

$$\Psi(z) \equiv \frac{\partial P(z)}{\partial z} = (\sigma - 1)p \tag{69}$$

Hence, if $\sigma > 1$ then $\Psi(z) > 0$ meaning that introducing actuarial non-neutrality gives agents an incentive to continue working.

Using equation (68), the budget constraint of the pension fund can be written as:

$$G(a^*)(\pi - \hat{z})^\sigma(\pi - z_L)^{1-\sigma}b + [1 - G(a^*)]\,(\pi - \hat{z})^\sigma(\pi - z_H)^{1-\sigma}b = \\ G(a^*)\tau q w(1 + z_L) + \tau w \int_{a^*}^{1} (a + z_H)\,\mathrm{d}G \tag{70}$$

Writing the budget constraint in terms of the fraction of low-skilled agents, $G(a^*)$, and high-skilled agents, $1 - G(a^*)$, is justified because when there is no heterogeneity in life expectancy retirement does not depend on ability anymore. As usual, the first (*second*) term on the left-hand side denotes pension benefits of the low-skilled (*high-skilled*) and the first (*second*) term on the right-hand side denotes pension contributions of the low-skilled (*high-skilled*) agents.

4.2 Consumption and retirement

The consumption decision and retirement decision have exactly the same form as in Section 3.4 (or Section 3.3). That is,

$$c_{nan} = c_{bev} + \frac{1}{1+\pi}\left[P_{nan} - P_{bev} - \frac{\Psi^2(z_{nan})\pi}{2\gamma}\right] \tag{71}$$

$$z_{nan} = z_{bev} + \frac{\Psi(z_{nan})\pi}{\gamma} \tag{72}$$

with $P(z)$ and $\Psi(z)$ defined by equation (68) and (69), respectively. Notice that:

$$\left.\frac{\partial z}{\partial \sigma}\right|_{\sigma=1} = \frac{\pi p}{\gamma} > 0 \tag{73}$$

Hence, a slight increase in the parameter σ indeed leads to later retirement. Consequently, the introduction of non-neutrality in the retirement decision can undo (at least to some extent) the distortionary effect of the social security tax τ. This result is comparable with what we had before, in the flexibility reform with uniform actuarial adjustment in combination with heterogeneous life spans. However, in that case, the pension scheme is still actuarially neutral at the aggregate level; the subsidy on continuing work is confined to high-skilled agents (with high life expectancy) at the expense of low-skilled agents (with low life expectancy). The current reform is different because now the pension scheme is actuarially non-neutral at the aggregate level. Therefore, the subsidy on work continuation can be captured by each ability group, irrespective of life expectancy.

4.3 Schooling

In the model, introducing actuarial non-neutrality has also positive effects on schooling. Because this reform stimulates to stay longer at the labour market, it pays for more agents to invest in acquiring skills. The following proposition summarizes this result formally.

Proposition 6. *The introduction of actuarial non-neutrality with respect to the retirement decision aimed at stimulating later retirement increases the fraction of high-skilled agents in the economy. That means,*

$$a^*_{nan} < a^*_{bev}$$

Proof. The cut-off point is determined by the condition $U_H(a^*) = U_L(a^*) \Rightarrow c_H(a^*) = c_L(a^*)$ where the subscript refers to non-actuarial neutrality. From equation (71) we can infer:

$$a^*_{nan} = a^*_{bev} - \frac{\Theta}{(1-\tau)w} \tag{74}$$

with Θ again defined as,

$$\Theta \equiv P(z_H) - P(z_L) - \frac{\Psi^2(z_H)\pi}{2\gamma} + \frac{\Psi^2(z_L)\pi}{2\gamma}$$

From equation (74), it follows:

$$\left.\frac{\partial a^*_{nan}}{\partial \sigma}\right|_{\sigma=1} = \frac{\partial a^*_{bev}}{\partial \sigma} - \frac{1}{(1-\tau)w}\frac{\partial \Theta}{\partial \sigma} \tag{75}$$

where we have used that $\Theta = 0$ if $\sigma = 1$. Using equations (68) and (69), we derive from the definition of Θ:

$$\left.\frac{\partial \Theta}{\partial \sigma}\right|_{\sigma=1} = [\ln(\pi - z_L) - \ln(\pi - z_H)]\,(\pi - \hat{z})b > 0 \tag{76}$$

Hence, $a^*_{nan} < a^*_{bev}$. □

4.4 Welfare effects

Like earlier, in case of flexible pension take-up with skill-dependent and uniform actuarial adjustment, the condition for a Pareto improvement is the following:

$$P_{nan} - P_{bev} - \frac{\Psi(z)^2\pi}{2\gamma} \geq 0 \tag{77}$$

with $\Psi(z)$ given by equation (69).

Now we have the following welfare result.

Proposition 7. *Introducing actuarial non-neutrality aimed at stimulating work effort makes high-skilled workers strictly better off. In addition, the reform is Pareto improving if and only if* $\tau > \hat{\tau}$*, with:*

$$\hat{\tau} = \frac{[1 - G(a^*)] \ln\left(\frac{\pi - z_L}{\pi - z_H}\right)}{\frac{w(1-q^2)\pi}{2\gamma} \ln\left(\frac{\pi - z_L}{\pi - z_H}\right) + G(a^*)\frac{qw\pi}{\gamma(\pi - z_L)} + [1 - G(a^*)]\frac{w\pi}{\gamma(\pi - z_H)}} \tag{78}$$

This equation has a unique solution.

Proof. See Appendix A.3. □

Introducing actuarial non-neutrality does not only stimulate labour supply and educational attainment, it also leads to a Pareto improvement if the tax rate is sufficiently high. The intuition for this result is the same as before, in the reform with uniform actuarial adjustment (see Section 3.4). The subsidy on promoting work effort provided by an actuarially non-neutral adjustment of pension benefits reduces the existing labour supply distortion related by the contribution tax rate. Hence, this reform increases the total efficiency of the economy and raises welfare of all individuals.

Figure 3: Neutral versus non-neutral actuarial adjustment

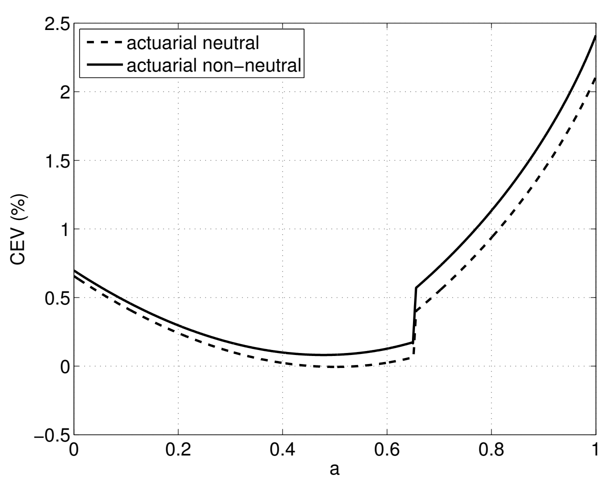


Figure 3 compares the welfare effects of uniform adjustment under actuarial neutrality (dashed line) and actuarial non-neutrality (solid line). Contrary to the analytical exposition discussed before, this figure is based on heterogeneous life expectancy. Except the tax rate, all remaining parameter values are the same as those used in the previous figures. In addition, we take $\sigma = 1.01$ for the actuarially non-neutral scenario. The tax rate is set such that uniform adjustment combined with actuarial neutrality is Pareto improving. This is the case for $\tau^* = 48\%$.

Notice that, given this tax rate, uniform adjustment of pension benefits combined with actuarial non-neutrality leads to strictly higher welfare gains for each a-individual compared to the situation in which uniform adjustment occurs in an actuarially-neutral way. Hence, by introducing actuarial non-neutrality in the pension scheme, it is possible to achieve a Pareto improvement for a lower contribution tax rate. Indeed, given our parameterization, it turns out that the tax critical rate is $\hat{\tau} = 44\%$.

5 Conclusion

In this paper, we have studied the intragenerational redistribution and welfare effects of a widely-applied pension reform, aimed at an increase in the opportunities for individual pension take-up. We have shown that moving to a pension scheme in which agents are free to choose the starting date of their pension income can induce a Pareto improvement. Such a reform entails the application of uniform actuarial adjustments of pension entitlements to ensure that high-skilled agents still benefit from intragenerational redistribution from short- to long-lived people. To compensate the low-skilled agents, a necessary condition is that the contribution rate is sufficiently high such that these people profit from an increase in the flat pension benefit. In addition, combining uniform adjustment with actuarial non-neutrality can further improve the reform.

In many real-world pension schemes, actuarial adjustment is indeed independent of individual characteristics, like life expectancy or skill level. In the Netherlands, for example, the idea of the recent first-pillar reform is to increase (*decrease*) the pension benefit by 6.5% when retirement is postponed (*advanced*) by one year. As such, the results of our paper might give a rationale for this kind of pension reforms based on uniform actuarial adjustment. However, in most countries the penalty rates of early retirement are still below the actuarially-neutral level (Queisser and Whitehouse, 2006). This means that there is still room to improve by going into the direction of complete actuarial neutrality or even beyond that level, as our analysis would suggest.

Our paper can benefit from a number of important extensions. One possibility is to extend the model with a Bismarckian pension scheme. Our benchmark scheme is of the Beveridgean type and characterized by inflexible pension take-up and life-time annuities.

Countries like the UK, the Netherlands and Denmark indeed follow this tradition. Other countries, like Germany, Italy and France are examples of the Bismarckian tradition in which pension benefits are linked in some way to former contributions. In general, these type of systems still contain intragenerational redistribution from short- to long-lived agents but have considerably less redistribution from rich to poor.

Other important elements to which we have not paid attention but that might be important when analysing pension flexibility, are the role of income effects or social norms. Especially in the short run, flexibility in the pension age could lead to only small changes in retirement behaviour if agents are used to retire on some socially accepted retirement age. In the long run, however, norms may change and the effects described in this paper may still apply. Although empirical research shows that the substitution effects modelled in this paper are more important for retirement decisions, income effects could also change retirement behaviour. For example, when actuarial adjustment is done in a uniform way income effects could prevent high-skilled (*low-skilled*) agents to postpone (*advanced*) retirement. To what extent these kind of issues would affect our main results, is left for future research.

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A Technical appendix

A.1 Proof of Proposition 3

Proof. Rewrite the budget constraint, equation (34), in the following way:

$$b(\bar{\pi} - \hat{z})\Phi = X \tag{A.1}$$

with:

$$\begin{aligned}
\Phi \equiv & \int_0^{a^*} \left[\frac{\pi - z_L}{\bar{\pi}_L - z_L} - \frac{(\pi - \bar{\pi}_L)\tau q w \pi}{\gamma(\bar{\pi}_L - z_L)^2} \right] \mathrm{d}G \\
& + \int_{a^*}^1 \left[\frac{\pi - z_H}{\bar{\pi}_H - z_H} - \frac{(\pi - \bar{\pi}_H)\tau w \pi}{\gamma(\bar{\pi}_H - z_H)^2} \right] \mathrm{d}G
\end{aligned} \tag{A.2}$$

$$X \equiv \tau q w \int_0^{a^*} \left[1 + \frac{(1-\tau) q w \pi}{\gamma} \right] \mathrm{d}G + \tau w \int_{a^*}^1 \left[a + \frac{(1-\tau) w \pi}{\gamma} \right] \mathrm{d}G \tag{A.3}$$

Note that in the benchmark $\Phi = 1$ while X is similar defined. From equation (A.1), we derive:

$$\left. \frac{\partial b_{bev}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\bar{\pi} - \hat{z}} \frac{\partial X_{bev}}{\partial \lambda} \tag{A.4}$$

$$\left. \frac{\partial b_{edu}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\bar{\pi} - \hat{z}} \left(\frac{\partial X_{edu}}{\partial \lambda} - X_{edu} \frac{\partial \Phi}{\partial \lambda} \right) \tag{A.5}$$

Hence,

$$\left. \frac{\partial b_{edu}}{\partial \lambda} - \frac{\partial b_{bev}}{\partial \lambda} \right|_{\lambda=0} = \frac{1}{\bar{\pi} - \hat{z}} \left(\frac{\partial X_{edu}}{\partial \lambda} - \frac{\partial X_{bev}}{\partial \lambda} - X_{edu} \frac{\partial \Phi}{\partial \lambda} \right) \tag{A.6}$$

From the definition of X we derive by applying Leibniz rule:

$$\begin{aligned}
\left. \frac{\partial X}{\partial \lambda} \right|_{\lambda=0} = & - \frac{\tau(1-\tau) w^2 (1-q^2) \bar{\pi}}{2\gamma} \frac{\partial a^*}{\partial \lambda} + \tau w \int_0^{a^*} \frac{(1-\tau) q w \bar{\pi} (a - \bar{a})}{\gamma} \mathrm{d}G \\
& + \tau q w \int_{a^*}^1 \frac{(1-\tau) w \bar{\pi} (a - \bar{a})}{\gamma} \mathrm{d}G
\end{aligned} \tag{A.7}$$

This implies:

$$\begin{aligned}
\left. \frac{\partial X_{edu}}{\partial \lambda} - \frac{\partial X_{bev}}{\partial \lambda} \right|_{\lambda=0} &= \frac{\tau(1-\tau) w^2 (1-q^2) \bar{\pi}}{2\gamma} \left(\frac{\partial a^*_{bev}}{\partial \lambda} - \frac{\partial a^*_{edu}}{\partial \lambda} \right) \\
&= \frac{\tau w (1-q^2) \bar{\pi}}{2\gamma} \frac{\partial \Theta}{\partial \lambda}
\end{aligned} \tag{A.8}$$

where we have used equation (41) in going from the first to the second line. Substituting this expression in equation (A.6) gives:

$$\left.\frac{\partial b_{edu}}{\partial \lambda} - \frac{\partial b_{bev}}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\bar{\pi} - \hat{z}}\left[\frac{\tau w(1-q^2)\bar{\pi}}{2\gamma}\frac{\partial \Theta}{\partial \lambda} - X_{edu}\frac{\partial \Phi}{\partial \lambda}\right] \tag{A.9}$$

We already know from equation (46) that $\frac{\partial \Theta}{\partial \lambda} < 0$. We still have to derive $\frac{\partial \Phi}{\partial \lambda}$. From the definition of Φ, we find:

$$\begin{aligned}\left.\frac{\partial \Phi}{\partial \lambda}\right|_{\lambda=0} = &\left[\frac{1}{\bar{\pi} - z_L} - \frac{\tau q w \bar{\pi}}{\gamma(\bar{\pi} - z_L)^2}\right]\int_0^{a^*}\left(\frac{\partial \pi}{\partial \lambda} - \frac{\partial \bar{\pi}_L}{\partial \lambda}\right)\mathrm{d}G \\ &+ \left[\frac{1}{\bar{\pi} - z_H} - \frac{\tau w \bar{\pi}}{\gamma(\bar{\pi} - z_H)^2}\right]\int_{a^*}^{1}\left(\frac{\partial \pi}{\partial \lambda} - \frac{\partial \bar{\pi}_H}{\partial \lambda}\right)\mathrm{d}G\end{aligned} \tag{A.10}$$

Notice:

$$\begin{aligned}\int_0^{a^*}\left(\frac{\partial \pi}{\partial \lambda} - \frac{\partial \bar{\pi}_L}{\partial \lambda}\right)\mathrm{d}G &= \int_0^{a^*}\left[\bar{\pi}(a-\bar{a}) - \frac{\int_0^{a^*}\bar{\pi}(a-\bar{a})\,\mathrm{d}G}{G(a^*)}\right]\mathrm{d}G \\ &= \bar{\pi}\int_0^{a^*}(a-\bar{a})\,\mathrm{d}G - \bar{\pi}\int_0^{a^*}(a-\bar{a})\,\mathrm{d}G \\ &= 0\end{aligned} \tag{A.11}$$

Along the same lines, we also have:

$$\int_{a^*}^{1}\left(\frac{\partial \pi}{\partial \lambda} - \frac{\partial \bar{\pi}_H}{\partial \lambda}\right)\mathrm{d}G = 0 \tag{A.12}$$

Hence, we have $\left.\frac{\partial \Phi}{\partial \lambda}\right|_{\lambda=0} = 0$. Therefore, $\frac{\partial b_{edu}}{\partial \lambda} < \frac{\partial b_{bev}}{\partial \lambda}$. □

A.2 Proof of Proposition 5

Proof of part i). Rewrite the budget constraint, equation (56), in the now usual way:

$$b(\bar{\pi} - \hat{z})\Phi = X \tag{A.13}$$

with:

$$\Phi \equiv \int_0^{a^*}\left[\frac{\pi - z_L}{\bar{\pi} - z_L} - \frac{(\pi - \bar{\pi})\tau q w \pi}{\gamma(\bar{\pi} - z_L)^2}\right]\mathrm{d}G + \int_{a^*}^{1}\left[\frac{\pi - z_H}{\bar{\pi} - z_H} - \frac{(\pi - \bar{\pi})\tau w \pi}{\gamma(\bar{\pi} - z_H)^2}\right]\mathrm{d}G \tag{A.14}$$

and X given by equation (A.3). Similar to skill-dependent actuarial adjustment, we again have:

$$\left.\frac{\partial b_{uni}}{\partial \lambda} - \frac{\partial b_{bev}}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\bar{\pi} - \hat{z}} \left(\frac{\partial X_{uni}}{\partial \lambda} - \frac{\partial X_{bev}}{\partial \lambda} - X_{uni} \frac{\partial \Phi}{\partial \lambda} \right) \tag{A.15}$$

Using equation (A.7), we obtain:

$$\begin{aligned} \left.\frac{\partial X_{uni}}{\partial \lambda} - \frac{\partial X_{bev}}{\partial \lambda}\right|_{\lambda=0} &= -\frac{\tau(1-\tau)w^2(1-q^2)\bar{\pi}}{2\gamma} \left(\frac{\partial a^*_{uni}}{\partial \lambda} - \frac{\partial a^*_{bev}}{\partial \lambda} \right) \\ &= \frac{\tau(1-\tau)w^2(1-q^2)\bar{\pi}(1-q)X(a^*_{uni} - \bar{a})}{2(\gamma - w + \tau w)(\gamma - qw + \tau qw)} \end{aligned} \tag{A.16}$$

Equation (A.15) therefore becomes:

$$\left.\frac{\partial b_{uni}}{\partial \lambda} - \frac{\partial b_{bev}}{\partial \lambda}\right|_{\lambda=0} = \frac{X}{\bar{\pi} - \hat{z}} \left[\frac{\tau(1-\tau)w^2(1-q^2)\bar{\pi}(1-q)(a^*_{uni} - \bar{a})}{2(\gamma - w + \tau w)(\gamma - qw + \tau qw)} - \frac{\partial \Phi}{\partial \lambda} \right] \tag{A.17}$$

What about the derivative $\frac{\partial \Phi}{\partial \lambda}$? From the definition of Φ above and applying Leibniz rule, we obtain:

$$\begin{aligned} \left.\frac{\partial \Phi}{\partial \lambda}\right|_{\lambda=0} &= \frac{\bar{\pi}}{\bar{\pi} - z_L} \left[1 - \frac{\tau q w \bar{\pi}}{\gamma(\bar{\pi} - z_L)} \right] \int_0^{a^*} (a - \bar{a})\, \mathrm{d}G \\ &\quad + \frac{\bar{\pi}}{\bar{\pi} - z_H} \left[1 - \frac{\tau w \bar{\pi}}{\gamma(\bar{\pi} - z_H)} \right] \int_{a^*}^1 (a - \bar{a})\, \mathrm{d}G \end{aligned} \tag{A.18}$$

Inserting equation (58) in this expression, gives:

$$\left.\frac{\partial \Phi}{\partial \lambda}\right|_{\lambda=0} = \underbrace{\frac{\gamma(\gamma - qw)}{(\gamma - qw + \tau qw)^2}}_{\Pi_L} \int_0^{a^*} (a - \bar{a})\, \mathrm{d}G + \underbrace{\frac{\gamma(\gamma - w)}{(\gamma - w + \tau w)^2}}_{\Pi_H} \int_{a^*}^1 (a - \bar{a})\, \mathrm{d}G \tag{A.19}$$

Let $\tau = 0$. Then we have $\Pi_H > \Pi_L$ implying that $\frac{\partial \Phi}{\partial \lambda} > 0$ for any possible cut-off point $0 < a^* < 1$. From equation (A.17) then follows $b_{bev} > b_{uni}$. Taking the other extreme, $\tau = 1$, we obtain $\Pi_H < \Pi_L$ so that $\frac{\partial \Phi}{\partial \lambda} < 0$ for any value $0 < a^* < 1$. This implies from equation (A.17) that $b_{bev} < b_{uni}$. Hence, there must be a tax rate τ^* such that $b_{uni} = b_{bev}$. □

Proof of part ii). If $a^*_{uni} - a^*_{bev} \to 0$, it follows from equation (A.16) that:

$$\left.\frac{\partial X_{uni}}{\partial \lambda} - \frac{\partial X_{bev}}{\partial \lambda}\right|_{\lambda=0} = 0 \tag{A.20}$$

Hence,

$$\left.\frac{\partial b_{uni}}{\partial \lambda}-\frac{\partial b_{bev}}{\partial \lambda}\right|_{\lambda=0}=-b_{uni}\frac{\partial \Phi}{\partial \lambda} \tag{A.21}$$

This derivative is zero if $\frac{\partial \Phi}{\partial \lambda}=0$. Note from equation (A.19) that this is satisfied if and only if $\Pi_H(\tau^*)=\Pi_L(\tau^*)$ which has a unique solution, given by equation (64). Hence, $b_{uni}\geq b_{bev}$ if and only if $\tau\geq\tau^*$. □

A.3 Proof of Proposition 7

Proof. Rewrite the budget constraint, equation (70), in the usual way:

$$b(\pi-\hat{z})\Phi=X \tag{A.22}$$

with:

$$\begin{aligned}\Phi\equiv\, & G(a^*)\left[(\pi-z_L)^{1-\sigma}-\frac{\tau q w(\sigma-1)\pi}{\gamma(\pi-z_L)^{\sigma}}\right]\\ & +[1-G(a^*)]\left[(\pi-z_H)^{1-\sigma}-\frac{\tau w(\sigma-1)\pi}{\gamma(\pi-z_H)^{\sigma}}\right]\end{aligned} \tag{A.23}$$

with X (again) defined by equation (A.3). The Pareto-improving condition is:

$$\begin{aligned}\Gamma &\equiv P_{nan}-P_{bev}-\frac{\Psi^2(z)\pi}{2\gamma}\\ &=(\pi-\hat{z})^{\sigma}(\pi-z)^{1-\sigma}b_{nan}-(\pi-\hat{z})b_{bev}-\frac{\Psi^2\left(z_{nan}\right)\pi}{2\gamma}\geq 0\end{aligned} \tag{A.24}$$

where for at least one a-individual this inequality should hold strictly. Suppose we start from a situation of actuarial neutrality ($\sigma=1$). Then we slightly increase σ and will see what happens with Γ. Hence, we derive:

$$\left.\frac{\partial \Gamma}{\partial \sigma}\right|_{\sigma=1}=(\pi-\hat{z})\frac{\partial b}{\partial \sigma}+X\ln(\pi-\hat{z})-X\ln(\pi-z) \tag{A.25}$$

From equation (A.22) it follows:

$$\frac{\partial b}{\partial \sigma}=\frac{1}{\pi-\hat{z}}\left(\frac{\partial X}{\partial \sigma}-X\frac{\partial \Phi}{\partial \sigma}\right) \tag{A.26}$$

Note that $\Phi = 1$ if $\sigma = 1$. Using equation (A.3), we have:

$$\begin{aligned} \left.\frac{\partial X}{\partial \sigma}\right|_{\sigma=1} &= -\frac{\tau(1-\tau)w^2(1-q^2)\pi}{2\gamma}\frac{\partial a^*}{\partial \sigma} \\ &= \frac{\tau w(1-q^2)\pi X}{2\gamma}\ln\left(\frac{\pi - z_L}{\pi - z_H}\right) > 0 \end{aligned} \tag{A.27}$$

where we have used equation (76) in going from the first to the second line. Note that this derivative is positive because $z_H > z_L$. Using definition (A.23), we can derive:

$$\begin{aligned} \left.\frac{\partial \Phi}{\partial \sigma}\right|_{\sigma=1} = \ & \ln(\pi - \hat{z}) - G(a^*)\left[\ln(\pi - z_L) + \frac{\tau q w \pi}{\gamma(\pi - z_L)}\right] \\ & - [1 - G(a^*)]\left[\ln(\pi - z_H) + \frac{\tau w \pi}{\gamma(\pi - z_H)}\right] \end{aligned} \tag{A.28}$$

Substituting equations (A.27) and (A.28) into equation (A.26) and inserting the resulting expression in equation (A.25) ultimately implies:

$$\begin{aligned} \left.\frac{\partial \Gamma}{\partial \sigma}\right|_{\sigma=1} = \ & G(a^*)X\left[\ln(\pi - z_L) + \frac{\tau q w \pi}{\gamma(\pi - z_L)}\right] + \frac{\tau w(1-q^2)\pi X}{2\gamma}\ln\left(\frac{\pi - z_L}{\pi - z_H}\right) \\ & + [1 - G(a^*)]\,X\left[\ln(\pi - z_H) + \frac{\tau w \pi}{\gamma(\pi - z_H)}\right] - X\ln(\pi - z) \end{aligned} \tag{A.29}$$

For high-skilled agents we have $z = z_H$, implying:

$$\begin{aligned} \left.\frac{\partial \Gamma}{\partial \sigma}\right|_{\sigma=1} &= G(a^*)\frac{\tau q w \pi X}{\gamma(\pi - z_L)} + [1 - G(a^*)]\frac{\tau w \pi X}{\gamma(\pi - z_H)} \\ &\quad + X\left[G(a^*) + \frac{\tau w(1-q^2)\pi}{2\gamma}\right]\ln\left(\frac{\pi - z_L}{\pi - z_H}\right) > 0 \end{aligned} \tag{A.30}$$

Hence, high-skilled workers are strictly better off when moving from the benchmark scheme to a scheme with actuarial non-neutrality. What about low-skilled people? For these agents, $z = z_L$ which gives:

$$\begin{aligned} \left.\frac{\partial \Gamma}{\partial \sigma}\right|_{\sigma=1} &= G(a^*)\frac{\tau q w \pi X}{\gamma(\pi - z_L)} + [1 - G(a^*)]\frac{\tau w \pi X}{\gamma(\pi - z_H)} \\ &\quad + X\left[\frac{\tau w(1-q^2)\pi}{2\gamma} - 1 + G(a^*)\right]\ln\left(\frac{\pi - z_L}{\pi - z_H}\right) \end{aligned} \tag{A.31}$$

Suppose that $\tau \to 0$. Then $\frac{\partial \Gamma}{\partial \sigma} < 0$ implying that low-skilled agents are worse off after the reform. If on the other hand $\tau \to 1$, then $z_L = z_H = 0$ so that the last term vanishes.

Therefore $\frac{\partial \Gamma}{\partial \sigma} > 0$ which means that low-skilled also benefit from the reform. We have $\frac{\partial \Gamma}{\partial \sigma} = 0$ if $\tau = \hat{\tau}$.

To prove that $\hat{\tau}$ is a unique solution, we have to show that the derivative $\frac{\partial \Gamma}{\partial \sigma}$ is monotonically increasing in τ. Rewrite:

$$\left.\frac{\partial \Gamma}{\partial \sigma}\right|_{\sigma=1} = XA \tag{A.32}$$

with,

$$\begin{aligned} A \equiv{}& G(a^*)\frac{\tau q w \pi}{\gamma(\pi - z_L)} + [1 - G(a^*)]\,\frac{\tau w \pi}{\gamma(\pi - z_H)} \\ &+ \left[\frac{\tau w(1-q^2)\pi}{2\gamma} - 1 + G(a^*)\right] \ln\left(\frac{\pi - z_L}{\pi - z_H}\right) \end{aligned}$$

Since $X > 0$ the necessary and sufficient condition for $\frac{\partial \Gamma}{\partial \sigma} \geq 0$ is $A \geq 0$. This implies that $\hat{\tau}$ is a unique solution if and only if A is monotonically increasing in τ. Taking the derivative of A with respect to τ gives, after some algebraic manipulations:

$$\begin{aligned} \frac{\partial A}{\partial \tau} ={}& \underbrace{\frac{w(1-q^2)\pi}{\gamma} \ln\left(\frac{\pi - z_L}{\pi - z_H}\right)}_{>0} + \underbrace{\frac{G(a^*) q w(\gamma - q w)}{(\gamma - q w + \tau q w)^2}}_{>0} + \underbrace{\frac{[1 - G(a^*)]\, w(\gamma - w)}{(\gamma - w + \tau w)^2}}_{>0} \\ &+ \underbrace{\left[1 - G(a^*) - \frac{\tau w(1-q^2)\pi}{\gamma}\right]}_{\Upsilon} \underbrace{\frac{w\pi}{\gamma}\left(\frac{1}{\pi - z_H} - \frac{q}{\pi - z_L}\right)}_{>0} \end{aligned} \tag{A.33}$$

If $q \to 1$ then $G(a^*) = 1$ which means $\Upsilon > 0$. We can further derive:

$$\frac{\partial \Upsilon}{\partial q} = -1 - \frac{(1-\tau) q w \pi}{\gamma} + \frac{2\tau q w \pi}{\gamma} \tag{A.34}$$

which is negative provided that $\gamma > 2\tau q w \pi$. Hence, if q declines Υ increases. Thus, $\Upsilon > 0$ for any value of q and thus $\frac{\partial A}{\partial \tau} > 0$. □