# Third-Degree Price Discrimination in an OLIGOPOLISTIC MARKET 

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#### Abstract

In this paper a discrete choice model is suggested which generates unambiguously lower prices, if oligopolists discriminate by price. In a setting of two groups of consumers and two firms this is due to a different ranking of the elasticity of demand of the two groups by the two firms. Here, this ranking results from switching costs. It is argued that firms will not choose price discrimination which lowers their profits, if firms are symmetric. However, with asymmetric firms price discrimination cannot always be prevented. In this case there is an incentive to use price discrimination and it enhances welfare.


# Third-Degree Price Discrimination in an Oligopolistic MARKET 

## 1. Introduction

One motivation for the present paper derives from the situation of deregulated electricity markets in Germany. Early foreign based entrants were substantially hampered by price matching policies of the local electricity companies. This was and is not the sole method of fending off own markets. Another substantial difficulty consists in overcoming the refusal of local suppliers to transport the contracted quantity of electricity to potential customers of the entrants at reasonable prices if at all. Given the focus of the present paper, we will concentrate on the price matching problem despite of the importance of other impediments of competition. Price matching is a form of price discrimination as the price of a customer receiving an outside offer is lower without any systematic relation to costs. The anticompetitive potential of price matching has been analyzed by several authors (e.g. by Edlin (1997)). It is therefore tempting to call for a strict ban on price discrimination in order to promote competition. Indeed this was called for by lawyers in this explicit context (e.g. Hübschle (1998)). This could be supported by the results on the impact of price discrimination on welfare in a monopolistic context. Such an impact is ambiguous in general, but a necessary condition for a positive effect of price discrimination is a positive impact on the aggregate demand (e.g. Varian (1989)). One might conjecture that in the case of electricity such an effect is unlikely. However, the results derived in a monopolistic context may not carry over to an oligopolistic one. Indeed price discrimination has an unambiguously positive impact on the welfare of consumers in the context of the model developed in this paper. Therefore banning price discrimination per se seems not a wise thing to do. We shall return to this argument at various stages of the paper. In any case, it is desirable to have an idea about the impact of price discrimination in an oligopolistic market before making a judgement on the desirability of such a ban.

The analysis of price discrimination has been restricted to monopolistic markets in the larger part of the literature (see e.g. the survey of Varian (1989)). Only quite recently, some studies have been provided which focus on the instrument of price discrimination in a competitive framework. Prominent examples of these are Holmes (1989) and Corts (1998). Holmes argues that the essence of the result in a monopolistic set up can be transferred to a oligopolistic one: Comparing with uniform prices for all customers, at least
one group of customers loses (has to pay a higher price) and another group of customers wins if firms are allowed to use third-degree price discrimination. Corts makes the point that this result is due to the fact that it is assumed that the elasticities of demand of the groups of customers is ranked identically by the firms. Such an assumption is certainly justified in many circumstances. In the eyes of each airline the elasticity of demand for business trips is smaller than the elasticity of tourists. But in other circumstances such an uniform ranking is not adequate. For example if the locations of firms are geographically different the customers at the own location have more inelastic demand than the customers at the location of the competitor. But the competitor will rank the elasticities of the same groups of customers just inversely. Indeed any form of brand loyalty can produce such a non uniform ranking among firms. Corts argues that in such a situation all prices may be higher or lower with price discrimination than without. Hence, one major conclusion of his work is that third-degree price discrimination can have quite different effects from those obtained in a monopolistic context.

This finding provides another motivation for the present paper. A discrete choice model is offered which shows that third-degree price discrimination results in lower prices for all customers under quite general circumstances. It should be stressed at this point that Corts (1998) does not provide any prediction about the direction of change in prices due to discrimination for a specific model. This finding generalizes some aspects of the work of Bester/Petrakis (1996) and Anderson/Leruth (1993). These authors, however, focus on instruments to achieve price discrimination when firms cannot tell which specific customer belongs to which group of customers without such instruments (coupons in the case of Bester/Petrakis and mixed bundling in the case of Anderson/Leruth). Therefore while providing useful insights on the value of such instruments their analyses distract from the pure case of third-degree price discrimination.. Our results also generalize some findings on discriminatory pricing in a spatial context as presented e.g. in Anderson/dePalma/Thisse (1992), pp 330. There uniform pricing is discriminatory (due to transportation costs) and yields lower equilibrium prices than mill pricing. While the logic is similar to the one in the present paper the spatial model does not fit the particularities of the electricity case well where transportation costs do not play a major role and due to a long standing relationship between customer and local provider make switching costs a more important factor. These authors state on p. 332:" It is tempting to conclude that spatial price discrimination would emerge as an equilibrium outcome..." In the context of our model this will turn out to be not the case.

The model to be presented in the next section is to some extent chosen to reflect characteristics of a segment of the electricity market. We concentrate on customers which have a quite inelastic demand of electricity. Therefore we choose a discrete choice framework. Each customer demands one unit unless the price surpasses her reservation price. Electricity companies have built up customer loyalty in the past. This loyalty varies in degree within the group of home market customers. The set up is general enough to cover any case where firms have built up customer loyalty and customers‘ demanded quantity is fixed. Each firm having built up loyalty with one group of customers defines the two groups of customers. If no price discrimination is allowed each firm can set only one price. If to the contrary price discrimination is allowed each firm may charge a price to members of the "own" group and a different one to members of the competitor's group. Note that in the context of electricity markets in Germany the type of a customer can easily be recognized by location. In general we assume that this distinction poses no problem.

To understand the announced result note that in a context without price discrimination a firm would like to set a high price in order to extract as much rent as possible from the most loyal customers. At the same time the firm would like to set a low price to attract less loyal customers of the competitor. An equilibrium price will balance these two motivations. With price discrimination the firm has two instruments each of which she can use according to the two motivations. In particular, the firm is not restrained in offering a low price to attract less loyal customers from her competitor. Given there is always an incentive to use this instrument the competitor has to lower the price to his loyal customers in order to limit the erosion of market share. Thus all prices are lower than without price discrimination. A major part of the paper is devoted to verify this intuitive argument.

Some remarks may be helpful to relate our analysis to Corts (1998). First of all, the general structure of the present model fits into the framework of Corts. There is one minor exception in that in our model the profit functions are not twice continuously differentiable in general. Corts's results say that the non-discriminatory equilibrium prices may have any relationship to the discriminatory ones. He shows that any pair of prices in the set "between" the best response functions of the discriminatory case can be an equilibrium outcome without discrimination for some profit functions. In particular, such prices can be higher than with discrimination. In our model we start with a specific class of profit functions and are able to show that for this fairly large class the above mentioned result of lower prices is obtained. These results are therefore consistent with Corts but not contained in his.

The paper is organized as follows. The next section will present the formal model. The following section will establish existence and characteristics of a price equilibrium in pure strategies without price discrimination. Then the same is provided for the case of price discrimination and both equilibria are compared. After this analysis two sections are devoted to the incentives for price discrimination from the point of view of firms, an aspect which is usually neglected in the related literature. The first of these concentrates on the symmetric set up used in the preceding sections while the second analyses the impact of asymmetries. In a symmetric context we will argue that discrimination is not likely to occur. A final section of comments concludes.

## 2. The model

There are two firms. Both of which produce a commodity using a constant returns to scale technology. For most of the paper we shall concentrate on the symmetric case. Hence, both firms produce under constant marginal costs. In this case we normalize these marginal costs to zero. Except for parts of the paper where we comment on the impact of asymmetries of costs between firms we shall assume zero marginal costs.

Each consumer demands one unit of the commodity or none. There is some reservation price $r$ which the same for all consumers. They can be identified as belonging to one of two groups. To fix ideas suppose that both groups are different in location. Unless stated otherwise we will again concentrate on the symmetric case: both groups have the same number of members. Each group consists of an atomless mass of consumers. This mass is normalized to one, if not stated otherwise. Consumers in each group are different in their attachment of the firm that is located in the same region. If a consumer located in one region considers buying the commodity from the firm in the other region she has to incur some costs, $s$. In line with most of the discrete choice literature a consumer located in region 1 obtains a utility level

$$
r-p_{1},
$$

if she purchases a unit from the firm in region 1 and has to pay $p_{1}$. and she obtains a utility level

$$
r-p_{2}-s,
$$

if she purchases a unit from the firm in region 2 and has to pay $p_{2}$. $s$ can be interpreted as transportation costs or switching costs. They may depend on contract length with the firm
in the own region or just preferences for this firm. $s$ is distributed on $[0,1]$ according to some continuously differentiable density function $f$. $F$ will denote the corresponding distribution function. The following assumption is imposed on $f$ :

Assumption 1: $f^{2}+(1-F) f^{\prime}>0$ and $f^{2}-(1+F) f^{\prime}>0$ on the support of $f$.
This assumption will guarantee that prices are strategic complements. It is slightly stronger than the requirement of logconcavity. It is satisfied e.g. for the class density functions

$$
f(x)=1-b / 2+b x
$$

if $-2<b<1 / 2$. This class contains the uniform distribution function ( $b=0$ ). Raising $b$ puts more weight on consumers with high switching costs.

To prepare the ground for the equilibrium analysis let us spell out the demand resulting from this set up. Denote prices which are relevant for consumers in region 1 by $p$ and those relevant for consumers in region 2 by $q$. More specifically, let $p_{1}$ be the price charged in region 1 by the firm in region 1 (firm 1) and let $p_{2}$ be the price charged in region 1 by the firm in region 2 (firm 2). Analogously let $q_{1}$ be the price charged in region 2 by the firm 1 and let $q_{2}$ be the price charged in region 2 by the firm 2. If price discrimination is not allowed we have $p_{i}=q_{i}, i=1,2$.

Let us concentrate first on customers in region 1 . If $p_{1} \leq p_{2}$ firm 2 will not attract any customer from region 1 and the demand for firm 1 is equal to 1 . If $p_{1}>p_{2}$ the optimal decision of a customer for firm 1 is characterized by

$$
p_{1} \leq p_{2}+s \Leftrightarrow p_{1}-p_{2} \leq s .
$$

Hence demand for firm 1 is $1-F\left(p_{1}-p_{2}\right)$ and demand for firm 2 is $F\left(p_{1}-p_{2}\right)$. Analogous reasoning applies to region 2 . This implies the following profit function for firm 1:

$$
\pi_{1}\left(p_{1}, p_{2}, q_{1}, q_{2}\right)=p_{1}\left(1-F\left(p_{1}-p_{2}\right)\right)+q_{1} F\left(q_{2}-q_{1}\right)
$$

and an analogous expression for firm 2.

## 3. Price equilibrium without price discrimination

Without price discrimination profits of firm 1 are

$$
\Pi_{1}\left(p_{1}, p_{2}\right)=p_{1}\left(1-F\left(p_{1}-p_{2}\right)+F\left(p_{2}-p_{1}\right)\right)
$$

Let us derive the best response function of firm 1. Note that $\Pi_{1}$ is continuous but not twice continuously differentiable at $p_{1}=p_{2}$. Note that this precludes a simple invocation of Corts's existence result. As we want to compare the equilibrium levels of prices with and without discrimination, we have to characterize the equilibrium prices anyway. To do this we therefore treat the cases $p_{1}<p_{2}$ and $p_{1}>p_{2}$ separately.

For $p_{1} \leq p_{2}: \Pi_{1}\left(p_{1}, p_{2}\right)=p_{1}\left(1+F\left(p_{2}-p_{1}\right)\right)$. Hence first order conditions read:

$$
\begin{equation*}
1+F\left(p_{2}-p_{1}\right)-p_{1} f\left(p_{2}-p_{1}\right)=0 \tag{1}
\end{equation*}
$$

At the boundary $p_{1}=p_{2}$ this implies $p_{1}=1 / f(0)$. We will therefore make the following
Assumption 2: $f(0)>0$.

This establishes that $p_{1}=1 / f(0)=p_{2}$ will be a point on the best response function under assumptions 1 and 2 . Close to the boundary we can evaluate the slope of the best response function. In our case this amounts to

$$
\frac{f\left(p_{2}-p_{1}\right)-p_{1} f^{\prime}\left(p_{2}-p_{1}\right)}{2 f\left(p_{2}-p_{1}\right)-p_{1} f^{\prime}\left(p_{2}-p_{1}\right)}
$$

Noting that $p_{1}=(1+F) / f$ it follows from assumption 1 that the second order conditions are fulfilled, that such a solution is unique, and that this expression is positive and smaller than 1. Therefore the solution of the expression of the first order condition will rise and at some point hit the line $p_{2}-p_{1}=1$ where all attainable demand from region 2 is attracted. Note that this line has a slope equal to one. So the solution of the first order condition spelled out above must cross this line. Let $p_{2}{ }^{*}$ characterize this intersection point. For $p_{2}>p_{2}{ }^{*}$ firm 1 one can afford to raise prices one to one with raising prices of firm 2. Hence, for $p_{2}$ $>p_{2}{ }^{*}$ the best response function is equal to $p_{1}=p_{2}-1$.

The following figure exhibits the best response function of firm 1 . The numbers in parentheses relate to the numbering of the first order conditions. Their graphs have been drawn linearly although they are not linear in general. As can be seen from the figure the best response function has a kink at the $45^{\circ}$ - line (compare the analytic expressions).


We assume that the reservation price $r$ is high enough to never matter in an equilibrium. Therefore for $p_{2}{ }^{*} \geq p_{2}>1 / f(0)$ the best response function is the implicit function defined by the first order condition above and for $p_{2}>p_{2}{ }^{*}$ it is sufficient to concentrate on its part $p_{1}=p_{2}-1$. We thus have a sufficient characterization of the best response function for $p_{1} \leq$ $p_{2}$ and can turn to the complementary case $p_{1} \geq p_{2}$. In this case the profit function reads

$$
\Pi_{1}\left(p_{1}, p_{2}\right)=p_{1}\left(1-F\left(p_{1}-p_{2}\right)\right) .
$$

First order conditions are

$$
\begin{equation*}
1-F\left(p_{1}-p_{2}\right)-p_{1} f\left(p_{1}-p_{2}\right)=0 \tag{2}
\end{equation*}
$$

Consider first $p_{2}=0$. Obviously profits are zero $p_{1}=0$ and for $p_{1}=1$, but positive for $0<$ $p_{1}<1$. Hence the first order conditions have a solution $0<p_{1}<1$. The slope of the implicit function defined by the first order condition is

$$
\frac{f\left(p_{1}-p_{2}\right)+p_{1} f^{\prime}\left(p_{1}-p_{2}\right)}{2 f\left(p_{1}-p_{2}\right)+p_{1} f^{\prime}\left(p_{1}-p_{2}\right)}
$$

It can again be easily verified that assumption 1 guarantees that the second order conditions are satisfied and that the slope strictly between zero and one. This implies that we do not have to worry that the best response function may intersect the line $p_{1}-p_{2}=1$. Note that the implicit function defined by the first order condition must intersect $p_{1}-p_{2}=$ 0 . At this intersection point we have again $p_{1}=1 / f(0)=p_{2}$ due to assumption 2 .

We thus have the following structure of the best response function of firm 1 . For $1 / f(0) \geq$ $p_{2}$ the best response function is defined by (2) and has a positive slope strictly smaller than 1. For $p_{2}{ }^{*} \geq p_{2} \geq 1 / f(0)$ the best response function is defined by (1) and has a positive slope strictly smaller than 1 . For $p_{2} \geq p_{2} *$ the best response function is $p_{1}=p_{2}-1$. The best
response function of firm 2 has an analogous structure. This implies first that the two best response functions do intersect at $p_{1}=1 / f(0)=p_{2}$ and that they cannot intersect a second time. Therefore we have established the following

Proposition 1: Under assumptions 1 and 2 the price game without price discrimination has a unique Nash equilibrium in pure strategies and is given by $p_{1}=1 / f(0)=p_{2}$.

Note that for $f(x)=1-b / 2+b x$ this implies $p_{1}=2 /(2-b)=p_{2}$. The more consumers have high switching costs (larger $b$ ) the higher is equilibrium price. This just reflects the fact that with comparatively small numbers of consumers with small switching costs it does not pay to attract customers from the competitor. Therefore the balance shifts toward the appropriation of rents of the more loyal customers.

Having established the benchmark case of no price discrimination we can now proceed to the case of price discrimination.

## 4. Price equilibrium with price discrimination

The analysis of price discrimination is much easier because we can treat the two groups of consumers separately. This is due to our implicit assumption that a firm can not price discriminate across consumers in her own region. We can therefore concentrate on one region. Let it be region 1 . Firm 1 has a profit from customers of its own region which amounts to

$$
p_{1}\left(1-F\left(p_{1}-p_{2}\right)\right) .
$$

Note that this profit function was already studied in the preceding section for the case $p_{1}>$ $p_{2}$. Hence for small $p_{2}$ the best response function starts at a value smaller than 1 and then increases with a slope less than one until it hits the boundary $p_{1}=p_{2}$. At such prices all customers of region 1 purchase from firm 1. If firm 2 increases her price the optimal response is to increase the own price by the same amount. This gives already the structure of the best response function of firm 1: For $p_{2} \leq 1 / f(0)$ the best response function is given by (2) and for $p_{2} \geq 1 / f(0)$ the best response function is $p_{1}=p_{2}$.

Turning to firm 2, this firm makes profit

$$
p_{2} F\left(p_{1}-p_{2}\right)
$$

in region 1. Obviously, the firm can only make a profit, if she chooses a price smaller than the price of firm 1. Note that from this observation the announced result already derives. As we want to establish existence and uniqueness of the equilibrium, however, we have to study the best response function of firm 2 as well.

The first order condition is

$$
\begin{equation*}
F\left(p_{1}-p_{2}\right)-p_{2} f\left(p_{1}-p_{2}\right)=0 \tag{3}
\end{equation*}
$$

From this it is immediately clear that for $p_{1}=0$ the best response is $p_{2}=0$. The slope of the implicit function defined by(3) is

$$
\frac{f\left(p_{1}-p_{2}\right)+p_{2} f^{\prime}\left(p_{1}-p_{2}\right)}{2 f\left(p_{1}-p_{2}\right)+p_{2} f^{\prime}\left(p_{1}-p_{2}\right)} .
$$

It follows from assumption 1 that this expression is positive but smaller than one. Hence the best response function starts at the origin and increases with a slope smaller than one until it hits the boundary $p_{1}-p_{2}=1$. At such prices she has attracted all customers in region 1 . Therefore for larger $p_{1}$ than corresponds to this intersection point the best response function coincides with $p_{2}=p_{1}-1$.


As all best response functions have a positive slope less or equal to one and at the intersection point can only be located at a price combination where the slope is less than one, there exists an equilibrium in pure strategies and it is unique.

As the equilibrium price vector in the case of no price discrimination is on the best response function of firm 1 where the solution to (2) hits the $45^{\circ}$-line and the best response
function of firm 2 is strictly below this point equilibrium prices with price discrimination are always smaller than without. We have thus established:

Proposition 2: Under assumptions 1 and 2 the price game with price discrimination has a unique Nash equilibrium in pure strategies. Moreover, all equilibrium prices are smaller than $1 / f(0)$.

The intuition given in the introduction is therefore verified. Note that we cannot make any strong welfare case for or against price discrimination using this model. Clearly consumers gain, if price discrimination is allowed. But due to the inelastic demand assumed on the level of individual consumers there is no efficiency gain. All consumers get their unit of the commodity. The decrease in prices is therefore a strict redistribution between firms and consumers. Note however, that if there would be a nondegenerate distribution of reservation prices such that the lower end of its support cuts into equilibrium level $1 / f(0)$, some consumers are deterred from consumption without price discrimination while they are not, if price discrimination occurs. In such a case there would be an efficiency gain by allowing price discrimination.

So far we have not considered switching costs when evaluating the welfare consequences. Note however that in the case of no discrimination no switching costs occur, while in the case of discrimination they do occur. Do consumers gain by discrimination if these costs are taken into account? The answer is yes. Note that consumers that remain loyal to a firm under discrimination do not bear any switching costs. They have to pay a lower price than without discrimination and so they gain. The remaining consumers have the choice between this same lower price and an even lower price of the competitor. As they choose to purchase from the competitor their utility must be higher taking the switching costs into account. Hence, nothing changes. Note also that it is not compelling that such switching costs should be taken into account in such an evaluation. If switching costs represent transportation costs or if they represent information costs in the case of switching they should be taken into account. But if they represent loyalty to a firm that was built up in the past and loyalty only serves as a barrier to the attention paid to another firm's offer there is no loss of well being, if a sufficient price advantage leads attention and switching. For these reasons we will ignore the switching costs in evaluating consumers' welfare and focus on prices as before.

On welfare grounds our result would be a major objection to a ban on price discrimination. While in this context price discrimination is desirable the incentives for firms to pursue such a strategy are not clear. This will be taken up in the next section.

## 5. Incentives to discriminate in the symmetric case

Due to the symmetry assumptions imposed so far all firms lose by discriminating by price. This raises the question as to why firms might pursue such a strategy. One answer could be that this is due to prisoners' dilemma situation. If one firm would gain by unilaterally switching to discrimination this would be the natural answer. But does a firm gain by such a move? The answer is certainly in the affirmative, if one firm can reasonably expect that the other firm sticks to her nondiscriminatory price $1 / f(0)$, as can easily be verified from the first order conditions. Alternatively, this is evident from the fact that the discriminating firm has more choice variables than before. Therefore profits can be increased (cp.Corts (1998)).

However, as long as we assume that prices can be easily adjusted (in line with most of the literature on price competition) such an expectation is hard to defend. It seems that a decision to implement a discriminatory strategy is less easily reversible than a pricing decision. This would suggest a sequential structure of decision making. After all the essential question of a firm deciding on whether or not to discriminate aims at the expected reaction of the other firm. This could be modeled in a three stage game, where one firm chooses whether to discriminate or not in the first stage, the other firm chooses discrimination versus non-discrimination in the second stage, and the simultaneous pricing is performed in the third stage depending on the discrimination policies of the two firms.

In such a setup we will see that any initiating firm has no incentive to use price discrimination. Let us first look at the third stage of such a game. To this purpose we have first to derive equilibrium prices for the case where one firm does discriminate while the other firm does not. Let the discriminating firm be firm 1. In this case firm 1's profit would be

$$
p_{1}\left(1-F\left(p_{1}-p_{2}\right)\right)+q_{1} F\left(p_{2}-q_{1}\right)
$$

and firm 2's profit would be

$$
p_{2}\left(1-F\left(p_{2}-q_{1}\right)+F\left(p_{2}-p_{1}\right) .\right.
$$

Note that firm 1 would never set a price $p_{1}<p_{2}$. Therefore we can concentrate on the case $p_{1} \geq p_{2}$. For such prices the first order conditions are:

$$
\begin{equation*}
1-F\left(p_{1}-p_{2}\right)-p_{1} f\left(p_{1}-p_{2}\right)=0 \tag{4}
\end{equation*}
$$

$$
\begin{gather*}
F\left(p_{2}-q_{1}\right)-q_{1} f\left(p_{2}-q_{1}\right)=0  \tag{5}\\
1+F\left(p_{1}-p_{2}\right)-p_{2} f\left(p_{1}-p_{2}\right)-F\left(p_{2}-q_{1}\right)-p_{2} f\left(p_{2}-q_{1}\right)=0 \tag{6}
\end{gather*}
$$

To obtain a solution subtract (4) and (5) from (6) to obtain

$$
2 F\left(p_{1}-p_{2}\right)+\left(p_{1}-p_{2}\right) f\left(p_{1}-p_{2}\right)-2 F\left(p_{2}-q_{1}\right)-\left(p_{2}-q_{1}\right) f\left(p_{2}-q_{1}\right)=0 .
$$

If $2 F(x)+x f(x)$ is a monotone function this equality yields $p_{1}-p_{2}=p_{2}-q_{1}$. We impose from now

Assumption 3: $2 f(x)+x f^{\prime}(x)>0$.

This is sufficient to guarantee monotonicity and is satisfied by the class of linear density functions given in section 2, at least for $b>-1$. Obviously it is also satisfied whenever $f$ is nondecreasing. Using this we can write (6) as

$$
1-2 p_{2} f\left(p_{1}-p_{2}\right)=0 .
$$

Dividing by 2 and subtracting the result from (4) gives

$$
\begin{equation*}
1 / 2-F\left(p_{1}-p_{2}\right)-\left(p_{1}-p_{2}\right) f\left(p_{1}-p_{2}\right)=0 \tag{7}
\end{equation*}
$$

The left hand side is positive for $p_{1}-p_{2}=0$ and negative for $p_{1}-p_{2}=1$. Due to assumption 3 there exists a unique solution of (7) strictly between these boundary values. Inserting back into (4) - (6) gives all prices. Hence, an equilibrium in prices exists for our case.

Can we make any statements about the equilibrium prices in relation to the ones obtained for the case of no discrimination? We will argue that all prices are lower in the present case. For this to show, it is sufficient to show that $p_{1}$ is smaller than $1 / f(0)$. To see this it is helpful to use a graph.

(c) represents the graph of the implicit function defined by (4). (a) represents that of the implicit function defined by

$$
1+F\left(p_{1}-p_{2}\right)-p_{2} f\left(p_{1}-p_{2}\right)=0 .
$$

Note that this is the best response function of firm 2 in the case of no discrimination. Compare this function with the best response function of firm 2 in the present case, defined by (6). It is immediate that the graph of the implicit function defined by (6), (b), must be below (a). From this it follows that any equilibrium value of $p_{1}$ must be below $1 / f(0)$.

Summarizing, we have established:
Proposition 3: Under assumptions 1 to 3 there exists a unique price equilibrium for the case that one firm discriminates while the other does not. All resulting prices are below the level of the case without any discriminating firms.

What does this tell us with respect to the profit of the discriminating firm? How does this profit compare with the one without any discrimination? Firm 1 can gain by charging a low price to region 2 customers. But firm 2 will react and reduce prices to weaken the erosion of market share. For that reason firm 1 will lose customers in region 1 to firm 2. So the balance seems not clear. Note, however, that due to the fact that the differences in prices satisfy $p_{1}-p_{2}=p_{2}-q_{1}$ firm 1 loses as many customers in region 1 as she gains in region 2. Given that according to proposition 3 all prices are lower the discriminating firm 1 loses by using this policy. Together with the result that firm 1 loses profits by discriminating if the other firms responds by discriminating as well, we have:

Proposition 4: In comparison with the case without any discrimination the discriminating firm loses profit.

This implies that in a setting where the decision to discriminate or not proceeds sequentially and the reactions in the price setting stage are taken into account no firm will choose to discriminate. Such a setting seems appropriate in the current context as the case of no discrimination is the status quo and changing this status quo needs one firm to think about the consequences of a decision to attract the rivals customers by lower prices.

To complete the characterization of the subgame perfect equilibrium of the three stage game, we still have to find the best response of the second firm if firm 1 chooses discrimination. We thus have to compare the profits of firm 2 in the set-up above in both situations. If firm 1 discriminates while firm 2 does not it follows from the preceding arguments that firm 2's profits are just equal to her price $p_{2}$. Denote the solution of (7) in price differences by $x^{m d}$. Then using (6) we have

$$
p_{2}=1 / 2 f\left(x^{m d}\right)
$$

for this case. Let us turn to the case with both firms discriminating. Firm 2 has the following profit:

$$
q_{2}\left(1-F\left(q_{2}-q_{1}\right)\right)+p_{2} F\left(p_{1}-p_{2}\right)
$$

As $p_{i}=q_{j}$ for $i \neq j$ this can be written as

$$
p_{1}-\left(p_{1}-p_{2}\right) F\left(p_{1}-p_{2}\right) .
$$

Recall the first order conditions characterizing the discriminating equilibrium, (2) and (3),

$$
\begin{gathered}
1-F\left(p_{1}-p_{2}\right)-p_{1} f\left(p_{1}-p_{2}\right)=0 \\
F\left(p_{1}-p_{2}\right)-p_{2} f\left(p_{1}-p_{2}\right)=0 .
\end{gathered}
$$

Subtracting (3) from (2) and denoting the price differences by $x$ gives

$$
\begin{equation*}
1-2 F(x)-x f(x)=0 . \tag{8}
\end{equation*}
$$

Under assumption 3 this equation has a unique solution. Let it be denoted by $x^{d}$. Therefore the profit of firm 2 in the case of discrimination by both firms can be written as

$$
\frac{1-F\left(x^{d}\right)}{f\left(x^{d}\right)}-x^{d} F\left(x^{d}\right) .
$$

Thus firm 2 gains by discriminating iff

$$
\frac{1}{2 f\left(x^{m d}\right)}<\frac{1-F\left(x^{d}\right)}{f\left(x^{d}\right)}-x^{d} F\left(x^{d}\right) .
$$

Using (8) this can be transformed into

$$
\frac{1}{2 f\left(x^{m d}\right)}<\frac{1-2 F\left(x^{d}\right)\left(1-F\left(x^{d}\right)\right)}{f\left(x^{d}\right)} .
$$

Note that the numerator of the right hand side is larger than $1 / 2$. This is due to the fact that $F(1-F)$ is always smaller than $1 / 4$ because $F$ cannot be equal to $1 / 2$ (consult (8)). Hence this inequality is satisfied if $f\left(x^{d}\right) \leq f\left(x^{m d}\right)$. Comparing (7) and (8) yields $x^{d}>x^{m d}$. Therefore this inequality holds for nonincreasing density functions. Unfortunately, it is not clear what will happen if density functions are increasing. While this is not entirely satisfactory we have the following result:

Proposition 5: If assumptions 1 to 3 are satisfied and the density function is nonincreasing, then firm 2 gains by discriminating, if the competitor does.

This completes the characterization of the subgame perfect equilibrium of the three stage game. As mentioned above such a setting seems appropriate in present context. However, in different contexts a simultaneous decision to discriminate or not may be more appropriate. Does the analysis tell us anything about the subgame perfect equilibrium in such a two stage framework? Proposition 4 implies that no discrimination is an equilibrium. But proposition 5 implies that discrimination by both firms is another equilibrium. If we start with the status quo of no discrimination, this second equilibrium does not seem to be very relevant. It could only obtain if one firm expects the other firm to discriminate. Given the structure of the problem such an expectation seems not appropriate. Certainly we cannot exclude the possibility in the static modeling framework used so far. If we consider a dynamic framework, the discrimination equilibrium becomes less likely. While we will not pursue this idea in great depth it seem clear that in a supergame context the discrimination equilibrium can be prevented by one firm by committing not to discriminate even if the other firm does. While this commitment is not credible in one period context, it is credible in a repeated game context, because the other firm will have an incentive to switch back to non-discrimination. Hence, also in a setting with simultaneous decisions to discriminate or not the discrimination equilibrium is not very likely.

Another form of preventing price discrimination to take place would be the announcement of practicing price matching. Such an announcement would be a credible threat as well. Note that in our context all consumers have some switching costs. Hence, the threat to use price matching will effectively prevent any customer to switch to the competitor. Therefore the competitor will have no incentive to make such offers.

It should be noted, however, that such threats could indeed provide full monopoly power to both firms. In section 3 it was assumed that firms compete for customers. If one firm announces that she will enter competition if the other firm does or that she will use price matching, the competitor has no incentive to try to attract any customers from the rival. Hence, both firms could charge monopoly prices which would in the context of our model amount to the common reservation price $r$. Therefore an instrument for preventing price discrimination can easily be transformed into one which eliminates competition at all.

This strong effect is, of course, due to the special structure of demand considered here. It is easy to monitor the behavior of the "own" clientele. If one current customer does not demand the commodity any more from her local supplier. it must be true that she was attracted to the rival and cannot be attributed to some stochastic fluctuation in demand. Hence, the threat to make offers to the competitor's customers if the competitor does carries low informational costs. Price matching policies on the other hand would always be effective, as the customer has an incentive to provide the necessary information to the firm.

Another special feature of demand are the exogenously given switching costs. Absent such switching costs and fully informed consumers the threats pointed out above do not have the effect of stifling all kinds of competition be it by uniform prices or by discriminating prices. Indeed, the latter possibility disappears completely. Therefore the implications of price matching pointed out above for the present context cannot apply to markets where switching costs do not play a major role.

But note that in the case of electricity shortly after deregulation such switching costs can be expected to be high, as customers have had a long history of business relations to the local provider. Any change of supplier may be seen as associated with many risks. This means that suppliers do not have to build up a special reputation and generate switching costs. In contrast to other nonregulated markets, where firms have to generate loyalty, they are already there. Moreover, within the regulatory constraints firms had full monopoly power, especially in terms of being protected against entry in their region. There exists by and large agreement that regulated price were quite high. Thus there is no central issue of
increasing prices. If the situation is seen in an infinitely repeated game the regulated price can serve as a focal point. This would hint at the possibility that such markets would not be competitive at all after deregulation, as long as new competitors do not enter. But entry could again be stifled by price matching policies.

As the positive aspects of price discrimination can not be expected in such a market and the negative ones (in the form of price matching) prevent competition it is again tempting to conclude that in such a framework all kinds of price discrimination should be prohibited. But this conclusion seems premature as well. The above arguments rely essentially on the fact that all firms lose, if competition occurs. This is certainly so in the above model. One major reason for this to be the case is the complete symmetry assumed so far. Therefore the next section will be devoted to the impact of asymmetries between firms.

## 6. Discrimination in an asymmetric world

To begin with suppose that switching cost are asymmetric. Consider an extreme case where all consumers in region 1 have switching costs of 1 . So the distribution of switching costs is degenerate concentrating all mass on the upper boundary. The distribution of consumers in region 2 is assumed to satisfy assumptions 1 and 2 . To make things as easy as possible assume to the contrary of what we have assumed so far that $1 / f(0)>r$.

It is now easy to verify that the equilibrium price without price discrimination is the reservation price $r$ for both firms. Hence, competition does not have any effect. To see this claim note that for $r \geq p_{1}>p_{2}$ demand for firm 1 is equal to one. The same is true for firm 2. Firm 1 only supplies to her "own" customers because of the higher price and firm 2 cannot attract any customer because of the high switching costs in region 1 . This gives the best response functions for $p_{1}>p_{2}$ : they are $p_{1}=r$ and $p_{2}=r$ respectively. For $r \geq p_{2}>p_{1}$ demand for firm 1 is $1+F\left(p_{2}-p_{1}\right)$ and demand for firm 2 is $1-F\left(p_{2}-p_{1}\right)$. The best response function in this domain coincides therefore with the best response functions analyzed in section 3 . From the discussion there it is easy to verify that the best response function of firm 2 hits the restriction $p_{2}=r$ in the domain $p_{2}>p_{1}$ and that the best response function of firm 1 in section 3 always implies a higher price than $r$. Therefore both best response functions have the following structure: Firm 1's best response function is just $p_{1}=$ $r$. Firm 2's best response function coincides with the one in section 3 until it intersects with $p_{2}=r$. Then $p_{2}=r$ is the characterization of the best response function of firm 2. Taken together: both best response functions intersect at $p_{1}=p_{2}=r$.

This picture changes considerably if price discrimination is allowed. As firm 2 cannot compete in region 1 , firm 1 can ask the reservation price $r$ from region 1 customers. But it can now compete with a different price in region 2 . Obviously the resulting equilibrium prices in region 2 correspond to those in section 4 . While firm 2 loses, firm 1 gains. Thus there is an incentive for firm 1 to discriminate by price. There is no way for firm 2 to credibly threaten to pay back in kind. Here consumers gain and a ban on price discrimination would destroy this possibility. Note that this situation is special. It is an interesting case nevertheless because local public utilities are not allowed to supply electricity to foreign areas in Germany. This corresponds to prohibitive switching costs. Private electricity companies are allowed to supply electricity anywhere.

Let us now turn to asymmetries in production costs. We will verify that for sufficiently large differences in costs there is an incentive for the low cost firm to compete be it in uniform prices or be it in discriminating ones. To this purpose let us assume that firm 1 bears a disadvantage in costs. Marginal costs of firm 1 are equal to $c>0$, while firm 2's cost are still normalized to zero. Obviously not every difference in costs will bring forward an incentive for the low cost firm. Competition will drive down prices. Only if the high cost firm is sufficiently disadvantaged competition will be less intensive and the low cost firm can attract sufficiently many customers to make up for the lower prices in general. As a formal validation of this claim calls for comparisons of profits, we use a special case of assumptions 1 and 2: the uniform distribution. In the following we will first look at the equilibrium in uniform prices and then in discriminating prices.

In the case of uniform prices profits of the firms for the uniform distribution are

$$
\begin{gathered}
\Pi_{1}\left(p_{1}, p_{2}\right)=\left(p_{1}-c\right)\left(1-p_{1}+p_{2}\right) \\
\Pi_{2}\left(p_{1}, p_{2}\right)=p_{2}\left(1-p_{2}+p_{1}\right)
\end{gathered}
$$

as long as $\left|p_{1}-p_{2}\right|$ is smaller than one. From this it is easy to verify that the equilibrium values for uniform prices are

$$
p_{1}=1+\frac{2 c}{3}, p_{2}=1+\frac{c}{3}
$$

if $c<3$. We shall restrict ourselves to this case. The low cost firm charges lower prices and obtains higher profits. Profits for firm 2 are $(3+c)^{2} / 9$ in this case. If this firm would be able to prevent competition she would obtain the reservation price $r$. Given that the mass
of consumers in one region is normalized to one this is also the profit. Hence, firm 2 will have an incentive to compete iff

$$
(3+c)^{2}>9 r
$$

If $r$ is large enough this inequality cannot be satisfied. Restricting our discussion as in sections 3 and 4 to the case where $r>1$, the cost disadvantage $c$ must be large enough to soften price competition and to ease an increase of market share for firm 2. As we have restricted $c$ to be smaller than 3 , an incentive to compete exists for sufficiently high costs $c$ $<3$, if $r<4$. Hence, in this case a threat of firm 1 to compete if firm 2 does so has no deterring effect.

Let us now consider the case of price discrimination. The response functions of firm 2 are those of section 4 adapted for the special case of an uniform distribution. For firm 1 things are a bit more complicated. Note that $p_{1}$ and $q_{1}$ should be larger than $c$. This implies that the best response function of firm 1 in region 1 is

$$
p_{1}\left(p_{2}\right)=\max \left(p_{2},\left(1+c+p_{2}\right) / 2, c\right) .
$$

The best response function for firm 2 in region 1 is

$$
p_{2}\left(p_{1}\right)=\max \left(p_{1}-1, p_{1} / 2\right) .
$$

From this it is easy to verify that both best response functions intersect at

$$
\begin{gathered}
p_{1}=\frac{2(1+c)}{3}, p_{2}=\frac{1+c}{3}, \text { if } c \leq 2 \text { and } \\
p_{1}=c, p_{2}=c-1, \text { if } c>2
\end{gathered}
$$

This implies that firm 1 will make no sales in region 1 , if $c>2$.

Let us now turn to region 2 . The best response function of firm 1 in region 2 is

$$
q_{1}\left(q_{2}\right)=\max \left(\left(c+q_{2}\right) / 2, c, q_{2}-1\right)
$$

and the best response function of firm 2 in region 2 is

$$
q_{2}\left(q_{1}\right)=\max \left(\left(1+q_{1}\right) / 2, q_{1}\right) .
$$

The intersection of both best response functions occurs at

$$
\begin{aligned}
q_{1}=\frac{1+2 c}{3}, q_{2} & =\frac{2+c}{3}, \text { if } c \leq 1 \text { and } \\
q_{1} & =q_{2}=c, \text { if } c>1
\end{aligned}
$$

For $c>1$, firm 1 cannot attract any customers from region 2 . Combining the results for both regions, firm 1 makes zero profits, if $c \geq 2$. In this case firm 1 has no potential to threaten to use price discrimination, if the competitor does.

Under which circumstances does the low cost firm 2 have an incentive to use price discrimination? It is a tedious exercise to verify that it never pays to use price discrimination, if $c \leq 1$. For $1<c<2$, profits of firm 2 amount to

$$
c+(1+c)^{2} / 9,
$$

if price discrimination is used and $(3+c)^{2} / 9$ if not. From this it follows that price discrimination allows for higher profits iff $c>8 / 5$. The same holds true for $2 \leq c \leq 3$.

One subtlety should be noted at this point. Prices do not have to decrease in all instances by discrimination. In fact, it is easy to verify that for $c>8 / 5$ firm 2's price in region 2 is higher than the price in the case of no discrimination. A tedious but simple exercise reveals that aggregate welfare increases nevertheless. This is largely due the fact that consumers in region 1 profit from lower prices.

Summarizing the case of asymmetric production costs, sufficient differences in costs provide an incentive to the low cost firm to compete in uniform prices or even more vigorously in discriminating prices. There is no credible threat available to the disadvantaged firm 1 to pay back in kind. In this case there is a clear efficiency gain, as more customers are supplied by a low cost firm. Given that deregulation is among other things intended to give efficient firms an higher incentive to compete for market share, an outright ban on price discrimination appears not sensible at all in this context. There is, however, still the possibility of price matching which can be an important hindrance to competition. This supports the view of e.g. Edlin (1997) that the potential of special instances of price discrimination to reduce competition should be judged in each case separately. If there is a candidate for cases of price discrimination where one should be skeptical about its value to enhance competition, price matching is a very good one.

## 7. Concluding remarks

One issue of the present paper has been to show that third-degree price discrimination can lead to unambiguously lower prices in an oligopolistic context. A crucial feature of the model presented here was the fact that the elasticity of demand of the two groups of customers is ranked differently by the two firms. It was suggested that this phenomenon always occurs if each firm has a "home" market. In this market customers are loyal to some extent to the corresponding firm. This may be due to switching costs. Such costs may arise from transportation costs, investments in information gathering about the quality of a firm's commodities, influence by advertising, length of the period that a long term contract may express, and the like. Another crucial assumption was that firms have no difficulty in identifying the group a specific customer belongs to while they do not have this possibility within such groups. This is nothing else than assuming circumstances that allow thirddegree price discrimination.

Given that structure, it is immediate that each firm is tempted to try to attract customers from the rival by charging to them a lower price than to their "own" customers. It has been shown that this incentive leads to lower prices for all customers because firms have to protect their home markets by reducing prices.

This leads to lower profits for both firms, if they are symmetric. For more or less symmetric firms the question arises as to why they would use such discriminating practices. This is the second main issue of the paper. It was argued that firms in a simultaneous decision making framework and understanding this situation could protect themselves by credibly announcing that they would not use price discrimination if the competing firms do so. In a sequential framework firms have no incentive to use price discrimination as they can only lose if they have the consequences on prices in mind. In such circumstances it may thus arise the case that price discrimination will not be practiced although it would enhance the well being of consumers.

It thus appears that a ban on price discrimination would not do much harm. However, it was argued in the preceding section that this picture changes once firms are allowed to be asymmetric. If one firm has a sufficient cost advantage she cannot be deterred to discriminate in prices. Therefore a ban on price discrimination in fact may harm efficiency in an asymmetric context.

Some aspects of the model presented here are special. It would be interesting to see how elastic aggregate demand would change the picture. As was noted in section 4 this would
not be expected, if elastic aggregate demand would result from a nondegenerate distribution of reservation prices. In general it may be conjectured that the result depends on the specific relationship between individual demand and switching costs. Another promising avenue to proceed is to allow for more firms. In that case there should be some asymmetry in switching costs or other characteristics. If not, price discrimination would result in a Bertrand situation with "foreign" prices at the common level of marginal costs. This certainly shows that the result of decreasing prices in the case of price discrimination would be robust to such extensions, but the question of the incentives to use this instrument can only be answered positively if there is some asymmetry indeed. As a special case of such asymmetric situations it would be interesting to see the impact of an independent entrant. A new firm has no loyal customers. This could be modeled by higher switching costs which the entrant has to overcome and lower switching costs which the incumbent firms have to overcome in order to attract customers from the entrant. These topics will be pursued in the near future.

## References

Anderson, S. P., de Palma, A., and J.-F. Thisse (1992): Discrete Choice Theory of Product Differentiation, MIT Press

Anderson, S. P. and L. Leruth (1993): Why firms may prefer not to price discriminate via mixed bundling, International Journal of Industrial Organization, vol. 11, 1, 49-61

Bester, H. and E. Petrakis (1996): Coupons and oligopolistic price discrimination, International Journal of Industrial Organization, vol. 14, 2, 227-242

Corts, K.S. (1998): Third-degree price discrimination in oligopoly: all-out competition and strategic commitment, Rand Journal of Economics, vol. 29, 2, 306-323

Edlin, A.S. (1997): Do guaranteed-low-price policies guarantee high prices, and can antitrust rise to the challenge, Harvard Law Review, vol. 111, 528-575

Holmes, T.J. (1989): The effects of third-degree price discrimination in oligopoly, American Economic Review, vol. 79, 244-250

Hübschle, W. (1998): Die kartellrechtliche Mißbrauchsaufsicht über Strompreisdifferenzierungen nach der Energiewirtschaftsrechtnovelle, Markt und Wettbewerb, JG. 48, 2, 146-155

Varian, H. (1989): Price discrimination, in Schmalensee, R. and R.D. Willig (eds.): Handbook of Industrial Organization, vol. I, ch. 10, 596-634

