Optimal Design of Split-award Auctions with Economies of Scale and Multiple Periods

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The rapid growth of donor funded markets for medicines, vaccines, and healthcare products for low-income countries builds the need for effective procurement mechanisms. In many instances, these markets are characterized by homogenous goods, economies of scale in production, and cost advantages of generic suppliers. Clearly, in the short term (i.e., one period procurement) consolidating volume to one supplier reduces procurement cost because the buyer benefits from the pooling effects. However, in case of multiple periods the buyer may want to prevent a monopolistic market from increasing prices (in the future) by contracting a second supplier today. In this case, the buyer forgoes some of the single sourcing benefits in the short term to ensure long-term low procurement cost. One way to achieve this long-term market health is a split-award auction.

We study the (repeated) procurement of a homogenous good in a two-period environment in the presence of economies of scale and possible market withdrawal of a supplier (i.e., the occurrence of a monopoly). The model consists of two suppliers, one more cost effective than the other, bidding for a variable share of a procurement contract, both in a first-price and a second-price auction setup. We identify the factors that influence the volume share that the buyer awards to each supplier and if the buyer should prefer an auction format. We find that in some situations the buyer can benefit from split-awards and reduce prices compared to single-sourcing and that the auction format can have an impact, too.

1 Introduction

"Auctions are used to award contracts for a variety of product and service requirements in the public (e.g., defense systems and minicipal services) and private sectors (e.g., input supply and franchising). These auctions can result in a sole-source award, in which a single producer provides all of the required production, or in a split award, in which production is divided between two or more firms."1 "The emergence of new global funding mechanisms such as the Global Alliance for Vaccines and Immunization (GAVI), the Global Fund for HIV/AIDS, Tuberculosis and Malaria, UNITAID, and large foundations, such as the Bill and Melinda Gates Foundation has resulted in rapid growth in the donor funded markets for medicines, vaccines and other health

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technologies in low income countries.\textsuperscript{2} This aggregating of monies has led to a better buyer position in the face of the suppliers and because of expired patents, the market for these medicines and vaccines is homogenous\textsuperscript{3}. With increasing procurement for such funds the buyers evermore have to justify their procurement mechanisms to the sponsors and therefore need to maximize the social welfare in form of achieving low prices for the procured medicines and vaccines. The problem we are addressing in this paper is relevant for the procurement of medicines and vaccines, because the market for these goods is shaped by generic suppliers who offer a low price and suppliers who operate in R&D and don't achieve a production cost as low as the generic producers. Our main research question is how a split between different suppliers could lead to a lower procurement-price in the long run (over multiple periods), even under the presence of economies of scale. We will look at both first- and second-price auctions, but will later focus on second-price auctions due to their feature of a bidding nash-equilibrium\textsuperscript{4}. In Section 2 the model for the procurement is presented, which relates to the initial setting with one generic supplier (low-cost) and one supplier that operates in R&D (high-cost, no learning effects included). Section 3 determines the bidding behavior of these suppliers in a setting where they are fully informed about each others costs and Section 4 will then cover the analysis of economical feasible splits between the suppliers, if there are any. An example with an explicit cost-function for this is then presented in section 5. We conclude in Section 6.

**Literature**

Procurement via split-award auctions is examined in a sizable amount of literature with different incentives and assumptions. \textit{Anton and Yao (1989)}, for example, study the cost differences between single-sourcing and a split-award under both full and asymmetric information for first-price auctions. For their research they assume that "dual source efficiency"\textsuperscript{5} has to be present to make a split-award profitable to the buyer. They also show that, in a full information setting, single-sourcing always dominates a split. \textit{Anton, Brusco, Lopomo (2010)}, "provide conditions under which the buyer and suppliers all benefit from a split-award format"\textsuperscript{6}, if asymmetric information is given. \textit{Alcalde and Dahm (2011)} show that,

\textsuperscript{2} Pibernik and Yadav, 2012, Unpublished working document on drug procurement for developing countries.

\textsuperscript{3} In this paper we are talking only about homogenous goods, of course there are medicines and vaccines without expired patents.

\textsuperscript{4} A bidding nash-equilibrium are bids that are mutual best responses, hence a change leads to a lower payoff.

\textsuperscript{5} Anton and Yao, 1992, "Coordination in Split Award Auctions", The Quarterly Journal of Economics Issue 107, p. 681.

under certain assumptions, minority representation can lower the expected costs for the buyer, which is quite similar to the model in this paper, where a high- and a low-cost-supplier are present, with the exception of scale economies. Their paper "On the Complete Information First-Price Auction and its Intuitive Solution"(2010) helps us with the auction theory for first-price auctions. Because from chapter 4 on we will focus on second-price auctions for our research, Gong, Li and McAfee (2010, 2012) serve as a starting point for the bidding stage as they cover a second-price auction setting with a high- and a low-cost supplier, an investment stage and a variable split. We slightly change the model and include economies of scale in place of pre-bidding investment. Perry and Sákovics (2003) also use a second-price split-award auction to procure a given homogenous good under asymmetric information, but as opposed to this work they make it a sequential auction with more than two suppliers and conclude that "secondary contracts can induce entry,[and] they may also reduce the expected price". If there is a entry fee for the bidding competition, Klotz and Chatterjee (1995) deduce that "splitting an award can result in lower expected procurement costs, even in a one-time procurement setting".

The difference between this work and the work of others is that we will define a very specific model consisten of two suppliers, one less cost efficient than the other, and economies of scale for the cost-functions of these suppliers. Other than in the mentioned literature we include repeated procurement over multiple periods under the risk of market resignation of the suppliers.

2 The model
A buyer wants to procure a certain amount of goods, normalized to one unit which is fully divisible, from two suppliers, \( i = \{ L, H \} \). We consider L as the low-cost supplier and H as the high-cost supplier, with cost functions \( c_i(\alpha) \) defined by the following properties

\[
\begin{align*}
c_L(\alpha) &< c_H(\alpha) \quad \text{for all } \alpha \in (0;1]; \\
c_i(0) &= 0; \\
c_L'(\alpha) &\equiv c_H'(\alpha) \quad \text{for all } \alpha \in [0;1]; \\
c_i'(\alpha) &< 0 \quad \text{for all } \alpha \in [0;1].
\end{align*}
\]

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(1) is straightforward the cost advantage of L compared to H and (2) states, that there is no cost associated with zero production, hence there is always an incentive to participate in the bidding contest. Our condition to set the economies of scale is (4), where (3) keeps both cost functions in the same shape, thus we have a constant difference between \( c_L \) and \( c_H \). The buyer has a reservation price \( r_b \). If any supplier submits a bid above \( r_b \), there will be no procurement. For simplicity, we consider

\[ r_b > c_i(\alpha) \quad \text{for all} \quad \alpha \in [0;1]. \]  

(5)

This assumption is no restriction, as reservation prices are commonly observed in procurement auctions. \( r_b \) can also be seen as a budget constraint.

The timeline of the bidding game is as follows. (i) First the buyer announces the reservation price \( r_b \) and the size of the split \( \alpha \), if there is a possibility of a split-award outcome.\(^9\) (ii) The bidders simultaneously submit their bids. (iii.a) In a first-price auction, bidders get paid their bids. The lowest bidder will be awarded the entire contract if \( \alpha = 1 \) or the \( \alpha \)-contract for \( \alpha < 1 \) and the high bidder will be awarded the (1-\( \alpha \))-contract. (iii.b) If the auction is a second-price auction, the lowest bidder gets paid the second-lowest bid, and the second-lowest bidder gets paid \( r_b \).

A bid is a function \( b_i \) with respect to \( \alpha \) and for \( i = \{L, H\} \). The buyer tries to set \( \alpha \) to minimize the procurement costs. The next section will focus on identifying the bidding strategies of the suppliers in different information- and auction-settings. Section 4 concentrates on finding the optimal \( \alpha \) for the buyer in an one or multi period procurement.

3 Bidding strategies

Given the model specifications, we can now analyse the bidding behavior of the suppliers. This section will analyse the different bidding strategies for first- and second-price auctions in both single-source and split-award environments under full information.

(A) Single-sourcing

First off, for \( \alpha = 1 \) the auction is equivalent to a standard auction for the first-price setting, and equal to a standard vickrey auction for the second-price setting. It is common auction theory knowledge that there is no dominant bidding strategy for a standard first-price auction. The intuitive solution is that the low-cost supplier will win the contest with a bid equal to the costs

\(^9\) \( \alpha \) could also be announced afterwards, but an uncertainty in \( \alpha \) would lead to a more complicated model and, in correspondence with the economies of scale, to higher bids of the suppliers (risk premium). Since the buyer wants to minimize the costs of the procurement this would be counterproductive.
of the high-cost supplier. For our second-price auction, we know that there is a dominant strategy for each participant $i$, which is bidding the true cost $c_i(1)^{10}$. This leads to the first lemma:

Lemma 1.

Let $p_1(\alpha=1)$ denote the price paid by the buyer in a first-price auction with $\alpha=1$ and $p_2(\alpha=1)$ the price in a second-price auction. Then, under full information,

$$p_2(\alpha=1) \leq p_1(\alpha=1)$$  \hspace{1cm} (6)

holds.

Proof.

It is sufficient to show, that the cost for the buyer in a first-price auction is higher or equal to the cost in a second-price auction. The price paid in a second-price auction is $p_2=c_H(1)^{11}$. The only case, in which this price is equal to the costs of the first-price auction, is the intuitive solution with $p_1=c_H(1)$, but due to the fact that this is no dominant strategy, bids can lead to a price $\tilde{p}_1 > p_1$. Therefore we see that $p_2 \leq p_1$. \hspace{1cm} \Box$

We conclude the price paid by the buyer, for a second-price auction:

$$p_2(\alpha=1)=c_H(1)$$

(B) Split-awards

Now consider $1/2 < \alpha < 1$, i.e. there is a split-award auction with a big contract for the lowest bidder and a smaller contract for the second-lowest/other bidder. As stated before $\alpha$ is announced before bids are submitted. From Gong, Li and McAfee (2010), we know that, in a second-price auction, "truthful bidding may no longer be an equilibrium"$^{12}$ and is only if

$$\alpha(c_H(\alpha)-c_L(\alpha)) \geq (1-\alpha)(rb-c_L(1-\alpha))$$  \hspace{1cm} (13)

because if the right-hand side is bigger than

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11 As mentioned earlier it is a weakly dominated strategy for the bidders to bid their true costs.
the left-hand side, the low-cost supplier could achieve a higher payoff by bidding higher than \(c_i(\alpha)\). For \(\alpha = 1\) this always holds. For \(\alpha < 1\) this condition may not be satisfied if \(\alpha\) or the cost differences of the suppliers are not sufficiently large.

The following proposition characterizes the bidding strategies that shape a nash-equilibrium for the second-price auction.

Proposition 1.\(^{14}\)

For a second-price auction with split-awards \((1/2 < \alpha < 1)\)

the bidding nash-equilibrium is:

\[
 b_i = \frac{1-\alpha}{\alpha} \cdot (rb - c_i(1-\alpha)) + c_i(\alpha)
\]

(7)

Proof.

To proof that the bids are a nash-equilibrium, we have to show that they are reciprocally best responses. We do this by showing that it is inefficient for the low-cost supplier to bid for the \((1-\alpha)\)-contract and for the high-cost supplier to bid for the \(\alpha\)-contract. First we consider the bid \(b_H(\alpha)\) of the high-cost supplier as fixed. To alter his position from the \(\alpha\)-contract to the \((1-\alpha)\)-contract, the low-cost supplier would have to bid \(\tilde{b}_L(\alpha) > b_H(\alpha)\) and would then achieve a payoff equal to

\[
\tilde{p}_L(\alpha) = (1-\alpha) \cdot (rb - c_L(1-\alpha))
\]

Compared to his previous payoff

\[
p_L(\alpha) = \alpha \cdot \left\{ \frac{1-\alpha}{\alpha} \cdot (rb - c_H(1-\alpha)) + c_H(\alpha) - c_L(\alpha) \right\}
\]

\[
p_L(\alpha) = (1-\alpha) \cdot (rb - c_H(1-\alpha)) + \alpha \cdot (c_H(\alpha) - c_L(\alpha)) \geq \tilde{p}_L(\alpha) \quad \text{15}.
\]

Therefore, the low-cost supplier has no incentive to change his bid.

Now consider \(b_L(\alpha)\) fixed. The high-cost suppliers' payment is

\[
p_H(\alpha) = (1-\alpha) \cdot (rb - c_H(1-\alpha))
\]

\[\text{http://capcp.psu.edu/conferences/May2010/papers/mcafeepaper.pdf.}\]

\[\text{14 This proposition is a modification of Proposition 1 from "Split-award Auctions with Investment" (Gong et al., 2010, p. 8). The difference is that we included the economies of scale with the cost-function } c_i(\alpha).\]

\[\text{15 This inequality holds because of the definition of } c_i \quad \text{[(1) and (3)].}\]
To change his position, H would have to bid \( b_H(\alpha) < b_L(\alpha) \), and would then be awarded the \( \alpha \)-contract and a payoff
\[
p_H(\alpha) = \alpha \left( c_L(\alpha) - c_H(\alpha) \right) < 0 \leq p_H(\alpha) .
\]

Accordingly, by changing his bid, the high-cost supplier is worse off and the bids \((b_L(\alpha), b_H(\alpha))\) are a nash-equilibrium. □

Proposition 1 shows that there is a nash-equilibrium for the second-price split-award auction and the bids only depend on the own costs incurred by the suppliers for \( \alpha \). The low-cost supplier will win the \( \alpha \) contract while the high-cost supplier obtains the \((1-\alpha)\) contract. \( b_i(\alpha) \) is decreasing in \( \alpha \), a bigger \((1-\alpha)\)-contract leads to a less aggressive bid, because the incentive to win is lower.

Proposition 2 provides the individual payoffs of the suppliers and the total price paid by the buyer.

Proposition 2.
\[
\begin{align*}
p_L(\alpha) &= \alpha \left( \frac{1-\alpha}{\alpha} \cdot (rb - c_H(1-\alpha)) + c_H(\alpha) - c_L(\alpha) \right) \quad \text{(8)} \\
p_H(\alpha) &= (1-\alpha) \cdot (rb - c_H(1-\alpha)) \quad \text{(9)}
\end{align*}
\]

The associated procurement-price paid by the buyer is
\[
pp(\alpha) = \alpha \left[ \frac{1-\alpha}{\alpha} \cdot (rb - c_H(1-\alpha)) + c_H(\alpha) \right] + (1-\alpha)rb \quad \text{(10)}
\]

Proof.

Straightforward. □

Next up is the first-price setup. This causes problems because, as discussed earlier, there is no stable equilibrium in a first-price auction. "It is well known among auction theorists that the first-price sealed-bid auction mechanism under complete information does not possess a pure strategy Nash equilibrium"\(^{16}\). What can be shown is that there are bids for each supplier that won't be undercut. The high-cost supplier is fully informed about the costs of the low-cost supplier, therefore he knows that he will lose the bidding contest, but being assured one share, he can maximze his profits by bidding \( b_H = rb \) and getting awarded the \((1-\alpha)\) contract for sure, no matter what the low-cost suppliers bid is.

Proposition 3.\textsuperscript{17}

In a first-price auction for an $\alpha$- and a $(1-\alpha)$-contract

the procurement-price for the buyer will not go below

\[
p_1 = \alpha \left[ \frac{1-\alpha}{\alpha} \left( rb - c_H (1-\alpha) \right) + c_H (\alpha) \right] + (1-\alpha) rb.
\]  
(11)

Proof.

The bidders are still risk neutral. We know that there is no stable bidding equilibrium in a first-price auction, but for a sufficiently large $\alpha$ the low-cost supplier won't bid below

\[
b_L (\alpha) = \frac{1-\alpha}{\alpha} \left( rb - c_H (1-\alpha) \right) + c_H (\alpha).
\]  
(12)

The high-cost supplier has an incentive to bid the reservation price $rb$, because by bidding $rb$ he gets awarded a bigger profit than by submitting a bid below $b_L$ (straightforward). For a decreasing $\alpha$ the high-cost supplier will bid more aggressively and the low-cost supplier will raise his bid because the incentive to win decreases, but due to the economies of scale and the associated cost uncertainty $L$ will raise his bid always more than $H$ will lower his bid. Therefore $p_1$ (11) is the lower bound for the price paid by the buyer, because any linear combination of bids for a lower $\alpha$ will be equal to or higher than $p_1$.

Lemma 2.

In a split-award auction with shares $\alpha$ and $(1-\alpha)$, $p_1(\alpha)$ is the procurement cost in a first-price and $p_2(\alpha)$ the procurement cost in a second-price auction. For $\alpha \in (1/2; 1]$

\[
p_1(\alpha) \geq p_2(\alpha).
\]  
(13)

Proof.

With

\[
p_2 = \alpha \left[ \frac{1-\alpha}{\alpha} \left( rb - c_H (1-\alpha) \right) + c_H (\alpha) \right] + (1-\alpha) rb
\]

and the lower bound for $p_1$ given as

\textsuperscript{17} Note that we can not use the "revenue equivalence theorem" here, because costs aren't independently drawn from the same distribution (see (1)) and this is a multiple unit auction.
Lemma 1 and Lemma 2 imply that a buyer should always prefer a second-price auction over a first-price auction in both single-sourcing and a split-award outcome when information about cost realizations is available amongst all suppliers. Furthermore there is no need to separate the single-source and the split-award cases, because the combined bidding functions are continuous on \((1/2;1)\]. This is easy to see, because for \(\alpha = 1\) the bidding function \((7)\) leads to the same outcome as in a single-source procurement (as discussed in 3.(A)). In the next section we will identify the conditions under which a split can benefit the price performance of the auction.

4 Split adjustment

With the model and its characteristics set up, this section will focus on the optimal choice of the split with regard to the procurement costs. With the results of section 3, all auctions will be second-price auctions.

(One period procurement)

When there is no repeated purchase and economies of scale are present, single-sourcing always dominates a split-award outcome, because the cost function \(pp(\alpha)\) is strictly decreasing in \(\alpha\). Without the need of minority representation, \(\alpha = 1\) will minimize the costs to the buyer and will therefore be optimal. This is a straightforward result, because in an economies of scale environment, no "dual source efficiency"\(^{18}\) is present, and therefore joint production costs are always higher than production costs in a single-source outcome, which leads the suppliers to raise their bids for \(\alpha < 1\). Therefore the buyer will choose \(\alpha = 1\) and the low-cost-supplier wins the contract.

(n-period procurement)

In a scenario of repeated procurement it can be of interest to keep more than one supplier in the market. The simplest reason would be to prevent a monopoly in which a supplier would raise his bid to the reservation price or above. Our model covers two suppliers, a low-cost

\[^{18}\text{Anton and Yao, 1992, "Coordination in Split Award Auctions", The Quarterly Journal of Economics Issue 107, p. 681.}\]
(generic) and a high-cost supplier (R&D). Another incentive beyond cost calculation would be to keep a supplier that does research and development for new vaccines. More reasons could be present, but below we will focus on cost calculation.

For a repeated procurement the model has to be adjusted to cover effects that can occur in such a setting.

**Model extension**

*In a multi period procurement setting, a supplier i will leave the market after period t (as a result, he will not participate in any auction in period t* > t) if his profits are smaller than \( r_p_i \). When there is only one supplier j left in period \( t+1 \), hence the market is a monopoly, the monopolistic supplier will adjust its bid in period t to \( b^{t+1}_j = r_b \). There is no possibility of re-entrance to the market.*

With the extended model we can now identify the conditions under which a split can be profitable to the buyer. Because the high-cost supplier will always be awarded the smaller share, we will disregard the reservation profit of the low-cost supplier and consider \( p_L(\alpha) \geq r_p_L \) satisfied for all \( \alpha \in (1/2; 1) \). The focus lies on \( r_p_H \), since the buyer may want to select a split that keeps \( H \) in the market and thereby prevent a monopolistic scenario. For example, in a two period procurement setting, the \( \alpha \) that makes the buyer indifferent between a split-award auction and a single-source auction, is the one that satisfies the equation

\[
c_H(1) + rb = 2 \left\{ \alpha \left[ \frac{(1-\alpha)}{\alpha} (rb - c_H(1-\alpha)) + c_H(\alpha) \right] + (1-\alpha)rb \right\} .
\]

For n-periods, this would be

\[
c_H(1) + (n-1)rb = n \left\{ \alpha \left[ \frac{(1-\alpha)}{\alpha} (rb - c_H(1-\alpha)) + c_H(\alpha) \right] + (1-\alpha)rb \right\} ,
\]

but to keep it simple, we will focus on the two period setting. The case for n periods is analogue.

The \( \alpha \) that satisfies (14) will be labeled \( \alpha_{\text{min}} \), because it is the smallest \( \alpha \) that makes a split-award economical feasible, hence it is a lower bound for all efficient splits. Any \( \alpha < \alpha_{\text{min}} \) would make the split more expensive than the sole-source procurement, due to the strict monotonicity of the procurement-price-function (10) for \( 1/2 < \alpha < 1 \).

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19 The low-cost supplier will always win the \( \alpha \)-contract, therefore, without loss of generality, we consider his reservation profit as always satisfied.

20 The left-hand side are the costs for a sole-source procurement, the right-hand side for the split-award outcome. The same goes for equation 15 respectively for n periods.
Since this is a multi period setting a $\alpha_{\text{min}}$ exists if and only if it grants the high-cost supplier a minimum profit of $r_{p_H}$, so that he does not exit the market after period 1, which would make a split-award auction in period 2 impossible and thereby (14) inexistent. The condition under which $\alpha_{\text{min}}$ holds is

$$r_{p_H} \leq (1-\alpha)(rb-c_H(1-\alpha)) \quad 21.$$ (16)

This also leads to the upper bound for possible splits, the $\alpha$ that makes (16) an equation, $\alpha_{\text{max}}$. As stated before, the procurement-price-function is strictly monotonically decreasing in $\alpha \in (1/2;1]$ and therefore $\alpha_{\text{max}}$ minimizes the price paid by the buyer. The following lemma will summarize the previous:

**Lemma 3**

Let $\alpha_{\text{max}}$ satisfy $r_{p_H} = (1-\alpha)(rb-c_H(1-\alpha))$ and $\alpha_{\text{min}}$ satisfy (14),

then an economically feasible split in a two period procurement exists if and only if

$$\alpha_{\text{min}} \leq \alpha_{\text{max}}.$$ (16)

For $\alpha_{\text{min}} < \alpha_{\text{max}}$ the price paid by the buyer while using $\alpha_{\text{max}}$ minimizes the procurement costs.

![Figure 1: procurement-price as a function of $\alpha$ with $\alpha_{\text{min}} < \alpha_{\text{max}}$](image)

21 The right hand side is the profit of the high-cost supplier for $\alpha$ (see also equation (8)).
The results of Lemma 3 are demonstratively shown in Figure 1. Over two periods the split at $\alpha_{\text{min}}$ would cost the buyer the same amount of money as in a sole-source procurement, thus, due to the strictly decreasing monotonicity of $pp(\alpha)$, for every $\alpha > \alpha_{\text{min}}$ the procurement-price is lower than in the sole-source setting. Because $\alpha_{\text{max}}$ is the upper bound for all possible splits it is also the optimal choice for the buyer. $\delta$ is the premium the buyer is willing to pay in order to keep the high-cost supplier in the market, which leads to a lower price in period two and a better overall price performance.

The existence of such an $\alpha_{\text{max}}$ is not assured and is conditional on the reservation price, the reservation profit and the cost-function of the high-cost supplier. For $\alpha_{\text{max}} < \alpha_{\text{min}}$ the premium $\delta$ would be higher than the actual cost savings in period two, which would make the split economical inefficient in comparison to a sole-source procurement.

In section 5 we will look at an explicit cost-function to calculate $\alpha_{\text{max}}$ and $\alpha_{\text{min}}$ and see how they are affected by the endogenous parameters.

5 Example
We define the cost-function $c_i(\alpha)$ as

$$c_i(\alpha) = \beta(1-\alpha) + \pi_i$$

for $\beta < (rb - \pi_H)$ and $rb > \pi_H > \pi_L$ (see also (1) – (4)).

![Figure 2: cost-functions for Example 1](image)

By inserting $c_i(\alpha) = \beta(1-\alpha) + \pi_i$ in (14) and solving for $\alpha$ we obtain $\alpha_{\text{min}} = 0.75$, which is a

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22 Note that $pp(\alpha)$ can be nonlinear.
remarkable result because \( \alpha^{\text{min}} \) is independent from \( rb, \pi_i, \) and \( \beta \). Economically speaking this means that a feasible split in a two period setting, if existent, can not be below 0.75. Second we have to solve \( rp_{hi} = (1 - \alpha)(rb - c_i(1 - \alpha)) \) for \( \alpha \) in order to obtain \( \alpha^{\text{max}} \). After inserting \( c_i(\alpha) \) this leads to a quadratic equation with \( a = \beta, \ b = (- \beta - rb + \pi_{hi}), \ c = (rb - \pi_{hi} - rp_{hi}) \) and the resulting \( \alpha_{1/2} \) (if not imaginary). If \( 1 > \alpha_i > \alpha^{\text{min}}, \alpha_i \) minimizes the procurement-price over the two periods.

We see that if unit costs are constantly decreasing, a feasible split in a two period procurement can lie between 0.75 and 1, which is a quite large margin considering this is only for two periods. The premium a buyer would pay to achieve the same costs as in a single-source procurement is \( \delta = 0.5(rb - \pi_{hi}) \), which is a straightforward result due to the definition of \( \alpha^{\text{min}} \). A higher difference between the single-source price and the reservation price leads to a higher premium paid by the buyer. \( \delta \) can also be seen as the upper bound for all possible premiums a buyer is willing to pay. As a result, for every \( \delta < \delta \) he is better off if the reservation profit of the high-cost supplier is still satisfied. The calculations of \( \alpha^{\text{min}} \) and \( \delta \) can be found in the appendix. Figure 3 shows the procurement-price function \( pp(\alpha) \) for this example with an \( \alpha^{\text{min}} < \alpha^{\text{max}} \).

![Figure 3: procurement-price function for the example with \( \alpha^{\text{min}} < \alpha^{\text{max}} \)](image-url)
6 Conclusion

This paper studies the circumstances under which dual-sourcing with the presence of economies of scale can benefit the overall price performance for the buyer. It is no surprise that it is buyer optimal to award the whole contract to a single supplier if the procurement is done over one period, due to the production efficiencies. The interesting case is the repeated procurement over two or more periods and as shown in Section 4 an economical feasible split can already exist when the procurement is done over two periods. This is a result of our setting where market resignation of a supplier is possible if his profits fall short. The existence of such a split depends on the shape of the suppliers cost-function and their cost-advantage as compared to the market-price (stated as the reservation price). In Section 5 it is shown that if unit costs are constantly decreasing, a feasible split can lie between 0.75 and 1.

It's easy to understand that a buyer has to be well informed about costs and market structure to decide whether a split is efficient or not. This could be improved by including more than two suppliers, therefore increasing the competition, which would lead to more aggressive bids and a better overall price performance. We also left out the possibility of re-entrance to the market.

7 Appendix

We calculate the $\alpha^{\text{min}}$ and $\delta$ for $c_i(\alpha) = \beta(1-\alpha) + \pi_i$ as in Example.

For $\alpha^{\text{min}}$ we use equation (14):

$$c_H(1) + rb = 2 \left( \alpha \cdot \left( \frac{1-\alpha}{\alpha} \cdot (rb - c_H(1-\alpha)) + c_H(\alpha) \right) + (1-\alpha)rb \right)$$

$$\pi_H + rb = 2 \left( \alpha \cdot \left( \frac{1-\alpha}{\alpha} \cdot (rb - \beta(1-(1-\alpha)) + \pi_H) \right) + \beta(1-\alpha) + \pi_H + (1-\alpha)rb \right)$$

$$\pi_H + rb = 2 \left( (1-\alpha)(rb - \beta\alpha - \pi_H) + \alpha (\beta(1-\alpha) + \pi_H) + (1-\alpha)rb \right)$$

$$\pi_H + rb = 2 \left( (1-\alpha)(2rb - \beta\alpha - \pi_H + \beta\alpha + \alpha\pi_H) \right)$$

$$\pi_H + rb = 2 \left( (1-\alpha)(2rb - \pi_H) + \alpha\pi_H \right)$$

$$\pi_H + rb = 4rb - 2\pi_H - 4\alpha rb + 4\alpha\pi_H$$

$$0 = 3(rb - \pi_H) - \alpha \cdot 4(rb - \pi_H)$$
\[ \alpha_{\text{min}} = \frac{3(rb-\pi_H)}{4(rb-\pi)} = \frac{3}{4} = 0.75 \]

For \( \delta \) we first insert 0.75 into (10) and obtain:

\[ pp(0.75) = \frac{1}{2}(rb+\pi_H) \]

\[ \rightarrow \delta = \frac{1}{2}(rb-\pi_H) \]

References


